الجامعة اللبنانية كلية العلوم الاقتصادية وادارة الاعمال	مباراة دخول للعام : ٢٠١٧-٢٠١٨ فرع إدارة الأعمال	السبت في ١٥ تموز ٢٠١٧
عدد المسائل: اربع	مسابقة في مادة الرياضيات المدة: ساعتان	الاسم: الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

In the following table, only one of the proposed answers to each question is correct.
Write down the number of each question and give, with justification, the corresponding answer.

N°	Questions	Answers		
		a	b	c
1	The equation $(e^x-1)(e^x+2)=0$ has	two real solutions	no real solutions	one real solution
2	The solution set of the inequality $x(-1+\ln x)<0$ is	$]0;+\infty[$	$]0;e[$	$]1;+\infty[$
3	f is a function defined over $]0;1[\cup]1;+\infty[$ as $f(x)=\dfrac{-1+\ln x}{(\ln x)^2}$. An antiderivative of f is F, where F(x) =	$\dfrac{\ln x}{x}$	$1+\ln x$	$\dfrac{x}{\ln x}$
4	The representative curve of the function f defined, over $\mathbb{R}$, as $f(x)=x+\dfrac{2e^x}{e^x+1}$ has at $+\infty$ an asymptote with equation	$y=x+2$	$y=x+1$	$y=x$

II- (4 points)

A restaurant proposes to its clients the following formula: a daily dish and the choice of one dessert (apple pie or ice-cream) with or without coffee.
A client might choose an apple pie, an ice-cream, or none of them. The client cannot choose both desserts. We notice that:

50% of clients choose ice-cream.
30% of clients choose apple pie.
20% of clients do not choose any dessert.
Out of the clients choosing ice-cream, 80% choose coffee.
Out of the clients choosing apple pie, 60% choose coffee.
Out of the clients not choosing any dessert, 90% choose coffee.

One client from the restaurant is randomly chosen and interviewed. Consider the following events:

G: «The client chooses ice-cream» T: «The client chooses apple pie»
N: «The client does not choose any dessert» C: «The client chooses coffee»

1) a- Calculate the probabilities $P(G\cap C)$ and $P(T\cap C)$.
   b- Verify that P(C) = 0.76 .
2) a- Verify that $P(\overline{C}\cap\overline{G}) = 0.14$.
   b- Knowing that the client does not choose coffee, calculate the probability that he does not choose ice-cream.
3) The price of an ice-cream is 4 000 LL, of an apple pie is 4 000 LL, and of coffee is 3 000 LL.
   Each client chooses one daily dish only of fixed price of 18 000 LL.
   Let X be the random variable that is equal to the sum, in LL, paid by a client in this restaurant.
   a- Verify that the four possible values of X are: 18 000 , 21 000 , 22 000 and 25 000.
   b- Prove that $P(X=22\,000)=0.22$ and calculate $P(X=25\,000)$.

III- (4 points)

On the first of January 2015, Nadim deposits in a bank a sum of x LL with an interest annual rate of 6% compounded yearly. In addition, on the first of January of each coming year, and after the capitalization of the interest, Nadim adds the amount of 1 800 000 LL to the account. Let $U_0 = x$ and, for every natural number n, let U_n be the amount in this account on the first of January of year (2015 + n).

1) For every natural number n, justify that $U_{n+1}=1.06\,U_n+1\,800\,000$.
2) For every natural number n, let $V_n = U_n + 30\,000\,000$.
   a- Verify that the sequence (V_n) is a geometric sequence whose common ratio should be determined. Express, in terms of x, the first term of (V_n).
   b- Express U_n in terms of x and n.
3) Calculate the value of x so that the amount in the account will be 197 245 852.8 LL on the first of January 2019.

IV- (8 points)

Let f be the function defined over $]0;+\infty[$ as $f(x)=x-1-2\ln x$. Denote by (C) its representative curve in an orthonormal system $(O;\vec{i},\vec{j})$.

1) a- Determine $\lim\limits_{\substack{x\to 0\\ x>0}} f(x)$, then deduce an asymptote to (C).
   b- Determine $\lim\limits_{x\to+\infty} f(x)$.
2) Calculate $f'(x)$ and set up the table of variations of function f.
3) Prove that the equation f(x) = 0 has exactly two roots 1 and α. Verify that $3.5<\alpha<3.52$.
4) Calculate f(5) and f(7), then draw (C).
5) Determine in terms of α, the area $A(\alpha)$ of the domain bounded by (C) and the x-axis.
6) Let g the function defined as $g(x) = \ln(-f(x))$.
   a- Determine the domain of definition of g.
   b- Set up the table of variations of g.