الجامعة اللبنانية
كلية العلوم الاقتصادية
وإدارة الأعمال

مباراة دخول – إدارة الأعمال

السبت ٢١ تموز ٢٠١٨

مسابقة في مادة الرياضيات　　الاسم:
المدة : ساعتان　　الرقم:

I- (4 points)

In the table below, only one among the proposed answers to each question is correct. Write the number of each question and give, **with justification**, the answer that corresponds to it.

N°	Questions	Answers a	Answers b	Answers c
1	If $C^2_{n^2+n}=1$, then n =	1	2	3
2	For all real numbers x, we have $\ln(1+e^{-x})-\ln(1+e^{x})=$	0	$-x$	x
3	The solution set of the inequality : $(e^x-1)(1-e^{-x})>0$ is:	$\mathbb{R}$	$]-\infty;1]$	$]-\infty;0[\cup]0;+\infty[$
4	(Δ) and (Δ') are two parallel lines. A, B, C and D are 4 points on (Δ). I and J are 2 points on (Δ'). Then the number of triangles formed by the 6 points A, B, C, D, I and J is:	16	C_6^3	$C_2^1\times C_4^2$

II-(4 points)

Consider three urns U, V and W. Urn U contains 3 red balls and 2 black balls, urn V contains 2 red balls and 3 black balls and urn W contains 3 red balls and 3 black balls.

A- A random experiment consists of drawing a ball from U: if the ball is red we put it into V and if it is black we put it into W then **at the end** we draw two balls: one ball from V and one ball from W.

Consider the following events:

- R : « the ball drawn from U is red»
- C : « the ball drawn from V is red and that drawn from W is also red »

1) Calculate the probability P(R), P(C/R) and verify that $P(C\cap R)=\frac{3}{20}$.

2) Prove that $P(C)=\frac{153}{700}$, then calculate $P\left(\overline{R}/C\right)$.

B- In this part, the balls of the urns U, V and W are all placed in an urn T.

Then we draw randomly and simultaneously three balls from T.

Let X be the random variable that is equal to the number of red balls obtained.

1) Calculate $P(X=0)$ and $P(X\le 1)$.

2) Calculate $P\left(X\le 2 / X\ge 1\right)$.

III- (4 points)

On a certain date, Fadi deposits in a bank an amount of 20 000 000 LL at an annual interest rate of 9% compounded monthly. Every month and after the compounding of interest, Fadi withdraws 300 000 LL to pay the rent of his apartment. For all integers $n\ge 0$, denote by u_n the amount of money that Fadi has in this bank after n months. Thus $u_0=20000000$.

1) Calculate u_1 then verify that $u_{n+1}=1.0075\,u_n-300\,000$.

2) For all integers $n\ge 0$, let (v_n) be the sequence defined as: $v_n=u_n-40000000$.

   a- Prove that (v_n) is a geometric sequence with common ratio 1.0075 and whose first term v_0 is to be determined.

   b- Prove that $u_n=20\,000\,000\times\left[2-(1.0075)^n\right]$.

   c- Prove that the sequence (u_n) is strictly decreasing.

3) The 9% annual interest rate proposed by the bank is not sufficient for Fadi in order for him to pay the rent of his apartment for 8 years. How much money does he still need? Justify.

IV- (8 points)

Part A

Let f be the function defined on IR as $f(x)=1-xe^{x-1}$.

1) Determine $\lim\limits_{x\to-\infty} f(x)$ and $\lim\limits_{x\to+\infty} f(x)$.

2) Show that $f'(x)=-(x+1)e^{x-1}$ and set up the table of variations of f.

3) Calculate f(1) then study, according to the values of x, the sign of f(x).

Part B

Let g be the function defined on IR as $g(x)=x+1+(1-x)e^{x-1}$.

Denote by (C) the representative curve of g in an orthonormal system of axes $(O;\vec{i},\vec{j})$.

1) a) Study, according to the values of x, the relative positions of (C) and (D).

   b) Determine $\lim\limits_{x\to-\infty} g(x)$.

   c) Show that the line (D) with equation $y=x+1$ is an asymptote to (C) at $-\infty$.

2) Show that $\lim\limits_{x\to+\infty} g(x)=-\infty$.

3) Show that $g'(x)=f(x)$ then set up the table of variations of g.

4) Determine the coordinates of the point A on (C) where the tangent (T) is parallel to (D).

5) (C) intersects the x-axis in two points with abscissas α and β where $\alpha<-1$ and $\beta>2$.

   a- Draw (D), (T) and (C).

   b- Denote by $A(\alpha)$ the area of the region limited by (C), x-axis and the two lines with equations $x=\alpha$ and $x=1$. Prove that $2<A(\alpha)<\frac{1}{2}\left(2+\frac{1}{e}\right)^2$.