مسابقة في مادة الرياضيات
المدة: ساعتان

الاسم:
الرقم:

I- (4 points)

A company produces and sells electronic devices. The annual profit is shown in the table below:

Year	2007	2008	2009	2010	2011	2012
Rank of the year x_i	1	2	3	4	5	6
Profit in millions LL y_i	62	73	100	102	120	135

1) Determine the coordinates of G, center of gravity of (x_i, y_i).
2) Determine an equation of $(D_{y/x})$, the regression line of y in terms of x.
3) Calculate the correlation coefficient r and give an interpretation of the value thus obtained.
4) Determine the percentage of increase in annual profit from 2007 till 2010.
5) Suppose that the above model remains valid for twenty years.
   Estimate the annual profit in 2014.
6) A second model of adjustment is given by $y = 10(2x + \ln x)$.
   If in reality, the annual profit in 2014 is 30% more than that in year 2011, which model is better? Justify.

II- (4 points)

Consider three urns U, V and W. Urn U contains 3 red balls and 2 black balls, urn V contains 2 red balls and 3 black balls and urn W contains 3 red balls and 3 black balls.

A- A random experiment consists of drawing a ball from U: if the ball is red we put it into V and if it is black we put it into W then **at the end** we draw two balls: one ball from V and one ball from W.

Consider the following events:

- R : « the ball drawn from U is red»
- C : « the ball drawn from V is red and that drawn from W is also red »

1) Calculate the probability P(R), $P\left(C/R\right)$ and verify that $P(C \cap R) = \frac{3}{20}$.
2) Prove that $P(C) = \frac{153}{700}$, then calculate $P\left(R/C\right)$.

B- In this part, the balls of the urns U, V and W are all placed in an urn T.

Then we draw randomly and simultaneously three balls from T.

Let X be the random variable that is equal to the number of red balls obtained.

1) Calculate $P(X = 0)$ and $P(X \leq 1)$.
2) Calculate $P\left(X \leq 2 / X \geq 1\right)$.

III- (4 points)

On a certain date, Fadi deposits in a bank an amount of 20 000 000 LL at an annual interest rate of 9% compounded monthly. Every month and after the compounding of interest, Fadi withdraws 300 000 LL to pay the rent of his apartment. For all integers $n \geq 0$, denote by u_n the amount of money that Fadi has in this bank after n months. Thus $u_0 = 20000000$.

1) Calculate u_1 then verify that $u_{n+1} = 1.0075\, u_n - 300\,000$.
2) For all integers $n \geq 0$, let (v_n) be the sequence defined as: $v_n = u_n - 40000000$.
   a- Prove that (v_n) is a geometric sequence with common ratio 1.0075 and whose first term v_0 is to be determined.
   b- Prove that $u_n = 20\,000\,000 \times \left[2 - (1.0075)^n\right]$.
   c- Prove that the sequence (u_n) is strictly decreasing.
3) The 9% annual interest rate proposed by the bank is not sufficient for Fadi in order for him to pay the rent of his apartment for 8 years. How much money does he still need? Justify.

IV- (8 points)

Part A - Let f be the function defined over $[0;+\infty[$ as $f(x) = \frac{1}{2}e^{2x} - 2e^x + x + 3$.

Denote by (C) the representative curve of f in an orthonormal system $(O;\vec{i},\vec{j})$.

Let (Δ) be the line with equation $y = 2$.

1) Determine $\lim\limits_{x \to +\infty} f(x)$. Calculate f(1) and f(1.5).
2) Prove that $f'(x) = (e^x - 1)^2$ and set up the table of variations of f.
3) a- Show that the equation $f(x) = 2$ has a unique solution α.
   b- Show that $\alpha \in \left]0.8\,;\,0.9\right[$.
4) Draw (Δ) and (C).
5) Calculate, in terms of α, the area of region bounded by (C), (Δ) and y-axis.

Part B - In what follows let $\alpha = 0.897$.

A factory produces toys. The total cost function is modeled as $f(x) = \frac{1}{2}e^{2x} - 2e^x + x + 3$.

(where x is in thousands of toys and f(x) is in millions LL) ; $x \in [0\,;100]$

1) Calculate the number of toys for which the cost is equal to 2 000 000 LL.
2) The revenue, in millions LL, is expressed by $R(x) = x$. (suppose all production is sold).
   a- Calculate, in LL, the price of one toy.
   b- Prove that the profit, in millions LL, is expressed as $P(x) = -\frac{1}{2}\left[(e^x - 2)^2 + 2\right]$, then deduce that the factory does not gain for any production.
3) Determine the level of production for which the factory achieves a minimal loss.