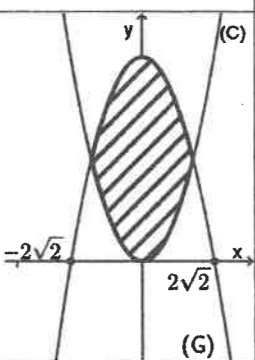


الاسم:  
الرقم:مسابقة في مادة الرياضيات  
المدة: ساعتان

عدد المسائل: أربع

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.  
- يستطيع المرشح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).**I- (4 points)**

Each one of the following questions has one and only one correct answer out of the proposed answers. Choose, with justification, the correct answer to each question.

N <sup>o</sup>	Question	A	B	C												
1	A box contains 60 bottles of juice. 10 % of the bottles of juice are orange juice. 4 bottles of juice are selected randomly and simultaneously from this box. How many selections contain exactly 3 bottles of orange juice among the 4 selected bottles?	3422	1080	20												
2	Each week, Jad arrives late to school 2 days out of 5 days and Jana arrives late to school 1 day out of 5 days. On a certain day, what is the probability that Jad arrives late to school and Jana arrives on time?	0.08	0.12	0.32												
3	(C) and (G) are the respective representative curves of two functions $f$ and $g$ defined, on $\mathbb{R}$ , as $f(x) = x^2$ and $g(x) = -x^2 + 8$ . The area of the shaded region is expressed as: 	$\int_{-2\sqrt{2}}^{2\sqrt{2}} (f(x) - g(x)) dx$	$\int_{-2}^2 (g(x) - f(x)) dx$	$\int_{-2}^2 (f(x) - g(x)) dx$												
4	The following table shows the probability distribution of a random variable X: <table border="1" data-bbox="192 1302 771 1375"> <tr> <td><math>x_i</math></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> <tr> <td><math>P(x_i)</math></td> <td>0.15</td> <td>0.55</td> <td>a</td> <td>0.05</td> <td>b</td> </tr> </table> where a and b are two real numbers. If the expected value $E(X) = 2.5$ , then	$x_i$	1	2	3	4	5	$P(x_i)$	0.15	0.55	a	0.05	b	a = 3 and b = 0.15	a = 0.1 and b = 0.15	a = 0.15 and b = 0.1
$x_i$	1	2	3	4	5											
$P(x_i)$	0.15	0.55	a	0.05	b											

**II- (4 points)**

A scientific electronic journal was launched in 2015 and is accessible only by subscription.

In 2015, the journal had 5 000 subscribed members.

Each year, 20 % of the subscribed members of the preceding year unsubscribe and 300 new members subscribe.

For all integers  $n \geq 0$ , denote by  $U_n$  the number of subscribed members for the year  $(2015 + n)$ .Thus,  $U_0 = 5 000$  and  $U_{n+1} = 0.8U_n + 300$ .1) a- Calculate  $U_1$ .

b- The annual subscription fee for a new member is 100 000 LL, whereas the renewal subscription fee is reduced by 10 %.

Calculate the total income of this journal collected from the subscription fees for the year 2016.

2) Consider the sequence  $(V_n)$  defined as  $V_n = U_n - 1500$  for all  $n \geq 0$ .a- Show that  $(V_n)$  is a geometric sequence whose common ratio and first term are to be determined.b- Verify that  $U_n = 3500 \times 0.8^n + 1500$ .c- Show that  $(U_n)$  is a strictly decreasing sequence.

d- Which year will the number of subscribers be less than 2 000 for the first time? Justify.

**III- (4 points)**

Given two urns U and V.

- Urn U contains two red balls and three green balls.
- Urn V contains four red balls and six green balls.

**Part A**

One ball from urn U and one ball from urn V are randomly selected.

- 1) Calculate the probability that the two selected balls are red.
- 2) Calculate the probability that the two selected balls have different colors.

**Part B**

A fair die is rolled. The die has six faces numbered 1 to 6.

- If the die shows 1 or 6, then two balls are randomly and simultaneously selected from urn U;
- otherwise, two balls are randomly and simultaneously selected from urn V.

Consider the following events:

E: "The die shows 1 or 6".

F: "The two selected balls are red".

1) a- Calculate the probability  $P\left(\frac{F}{E}\right)$  and deduce that  $P(E \cap F) = \frac{1}{30}$ .b- Calculate  $P(F)$ .

2) The two selected balls are red. Calculate the probability that the die neither shows 1 nor 6.

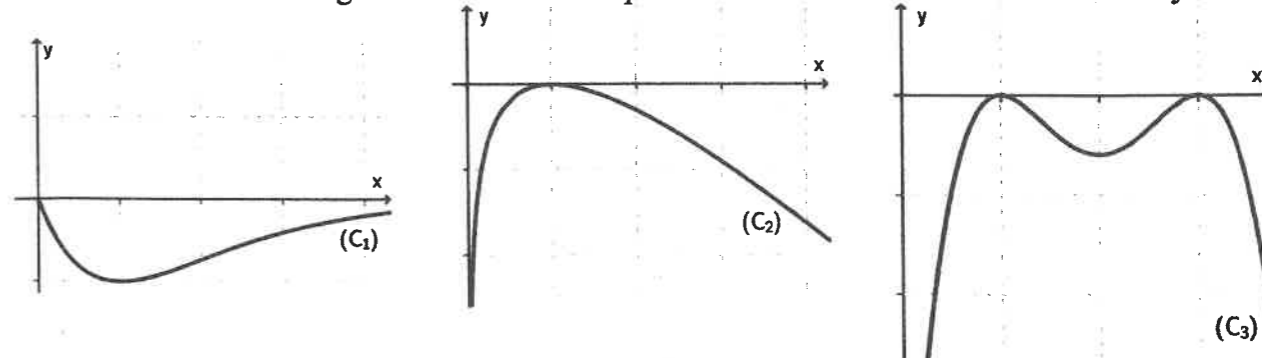
3) Let X be the random variable equal to the number of the selected red balls.

a- Verify that  $P(X = 0) = \frac{29}{90}$ .

b- Determine the probability distribution of X.

**IV- (8 points)**Consider the function  $f$  defined over  $] -\infty, +\infty[$  as  $f(x) = 3 - xe^{1-x}$ .Denote by (C) its representative curve in an orthonormal system  $(O; \vec{i}, \vec{j})$ .1) Determine  $\lim_{x \rightarrow -\infty} f(x)$  and calculate  $f(-1)$ .2) a- Show that the line (d) with equation  $y = 3$  is an asymptote to (C).b- Study, according to the values of  $x$ , the relative positions of (C) and (d).3) Verify that  $f'(x) = (x-1)e^{1-x}$  and set up the table of variations of  $f$ .4) The line (D) with equation  $y = 2.5$  intersects (C) at two points of abscissas  $\alpha$  and  $\beta$  with  $0.22 < \alpha < 0.24$ . Show that  $2.67 < \beta < 2.69$ .

5) Draw (C), (d) and (D).

6) Calculate the area of the region bounded by (C), (d) and the two lines with equations  $x = -1$  and  $x = 1$ .7) Consider the function  $h$  defined as  $h(x) = \ln(3 - f(x))$ .a- Verify that the domain of definition of  $h$  is  $]0, +\infty[$ .b- Determine  $\lim_{x \rightarrow 0} h(x)$  and  $\lim_{x \rightarrow +\infty} h(x)$ . Calculate  $h(1)$ .c- Which of the following three curves is the representative curve of the function  $h$ ? Justify.d- Solve the equation  $h(x) = -\ln 2$  then solve the inequality  $h(x) > -\ln 2$ .