الجامعة اللبنانية
كلية العلوم الاقتصادية
وإدارة الأعمال

مباراة دخول
إقتصاد

السبت ٢٠ تموز ٢٠١٩

عدد المسائل: أربع | مسابقة في مادة الرياضيات | الاسم:
المدة: ساعتان | الرقم:

ملاحظة: - يسمح باستعمال آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.
- يستطيع المرشّح الإجابة بالترتيب الذي يناسبه (دون الالتزام بترتيب المسائل الواردة في المسابقة).

I- (4 points)

A study was conducted on the number of inhabitants of a village.
The table below shows the number of inhabitants (in thousands) of the village, on January 1st of each year from 2010 to 2015.

Year	2010	2011	2012	2013	2014	2015
Rank of the year: x_i	1	2	3	4	5	6
Number of inhabitants (in thousands): y_i	10.5	11.5	12.9	14.5	15.4	16.9

1) Calculate the coefficient of correlation r and interpret the obtained value.
2) Determine an equation of the regression line $(D_{y/x})$, of y in terms x.
3) Calculate the percentage increase in the number of inhabitants from 2010 till 2015.
4) Assume that the above model remains valid during the period from 2010 till 2025.
   a- Will the number of inhabitants of the village exceed 30 000 in a certain year during this period? Justify.
   b- The number of apartments in this village is 5 000 and each apartment can accommodate an average of 5 people. In which year will the apartments of this village fail to accommodate the inhabitants of the village for the first time? Justify.

II- (4 points)

Given two urns U and V.
- Urn U contains two red balls and three green balls.
- Urn V contains four red balls and six green balls.

Part A

One ball from urn U and one ball from urn V are randomly selected.
1) Calculate the probability that the two selected balls are red.
2) Calculate the probability that the two selected balls have different colors.

Part B

A fair die is rolled. The die has six faces numbered 1 to 6.
- If the die shows 1 or 6, then two balls are randomly and simultaneously selected from urn U;
- otherwise, two balls are randomly and simultaneously selected from urn V.

Consider the following events:
E: "The die shows 1 or 6".
F: "The two selected balls are red".

1) a- Calculate the probability $P(F/E)$ and deduce that $P(E \cap F) = \frac{1}{30}$.
   b- Calculate P(F).
2) The two selected balls are red. Calculate the probability that the die neither shows 1 nor 6.
3) Let X be the random variable equal to the number of the selected red balls.
   a- Verify that $P(X = 0) = \frac{29}{90}$.
   b- Determine the probability distribution of X.

III- (4 points)

A scientific electronic journal was launched in 2015 and is accessible only by subscription.
In 2015, the journal had 5 000 subscribed members.
Each year, 20 % of the subscribed members of the preceding year unsubscribe and 300 new members subscribe.
For all integers $n \geq 0$, denote by U_n the number of subscribed members for the year $(2015 + n)$.
Thus, $U_0 = 5\,000$ and $U_{n+1} = 0.8U_n + 300$.

1) a- Calculate U_1.
   b- The annual subscription fee for a new member is 100 000 LL, whereas the renewal subscription fee is reduced by 10 %.
   Calculate the total income of this journal collected from the subscription fees for the year 2016.
2) Consider the sequence (V_n) defined as $V_n = U_n - 1500$ for all $n \geq 0$.
   a- Show that (V_n) is a geometric sequence whose common ratio and first term are to be determined.
   b- Verify that $U_n = 3500 \times 0.8^n + 1500$.
   c- Show that (U_n) is a strictly decreasing sequence.
   d- Which year will the number of subscribers be less than 2 000 for the first time? Justify.

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IV- (8 points)

Consider the function f defined over $[0, +\infty[$ as $f(x) = 3 - xe^{1-x}$.
Denote by (C) its representative curve in an orthonormal system $(O; \vec{i}, \vec{j})$.

Part A

1) a- Show that the line (d) with equation y = 3 is an asymptote to (C).
   b- Show that (C) is below (d) for all $x > 0$.
2) Verify that $f'(x) = (x-1)e^{1-x}$ and set up the table of variations of f.
3) The line (D) with equation y = 2.5 intersects (C) at two points of abscissas α and β with $0.22 < \alpha < 0.24$. Show that $2.67 < \beta < 2.69$.
4) Draw (C), (d), and (D).

Part B

In what follows, take α = 0.23 and β = 2.68.

A company produces and sells a certain type of watches.
The average cost of production, in millions LL, is modeled as $\overline{C}(x) = 3 - xe^{1-x}$ where x is the number of produced watches, in hundreds, with $0 < x \leq 4$.

1) Calculate $\overline{C}(2)$ and interpret economically the obtained value.
2) Denote by $C_T(x)$ the total cost of production in millions LL.
   Express $C_T(x)$ in terms of x.
3) Determine, graphically, the number of watches produced so that the average cost is minimum.
4) Every watch is sold for 31 250 LL and only 80 % of the production is sold.
   a- Show that the profit P(x), in millions LL, is modeled as $P(x) = x[2.5 - \overline{C}(x)]$.
   b- Determine the number of watches sold so that the company achieves a gain.
   c- The average profit $\frac{P(x)}{x}$, in millions LL, is denoted by $\overline{P}(x)$.
   Show that $\overline{P}(x)$ is maximum for x = 1 and calculate this maximum. Interpret the obtained result economically.