LEBANESE UNIVERSITY
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Entrance Exam 2019 - 2020 *Mathematics* *Duration : 3 hours*
The distribution of grades is over 50 *July , 2019*

Exercise 1 **(6 points)**

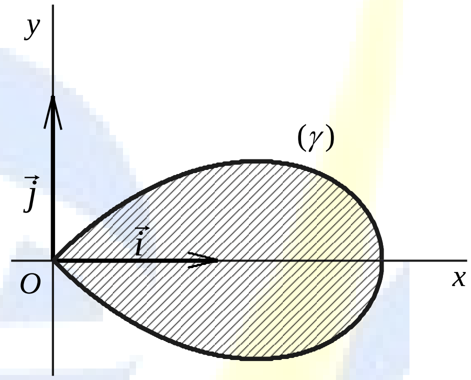
The plane is referred to an orthonormal system .

Consider the curve (γ) of equation $x^3 - 2x^2 + xy^2 + 2y^2 = 0$.

1- a) Prove that the equation of (γ) is equivalent to $y^2(2+x) = x^2(2-x)$.
Deduce that the set of abscissas of the points of (γ) is the interval $I =]-2\ ;\ 2]$.

b) Prove that the axis of abscissas is an axis of symmetry of (γ) .

2- The adjacent figure shows the part of (γ) , in $[0\ ;2]$, in an orthonormal system $(O\ ;\vec{i}\ ,\ \vec{j}\ )$.



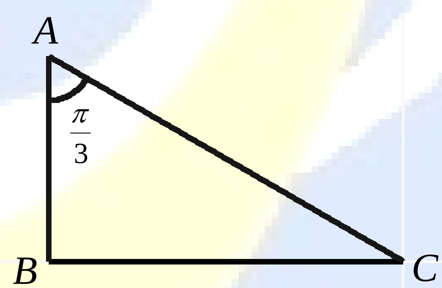
a) Verify that $y^2 = -x^2 + 4x - 8 + \dfrac{16}{x+2}$.

b) Calculate the volume V of the solid generated by the rotation of the shaded domain bounded by (γ) about the axis of abscissas .

c) If $\|\vec{i}\| = \|\vec{j}\| = 2\,cm$, determine V in cm^3 .

Exercise 2 **(10 points)**

Consider in an oriented plane , a triangle ABC right at B , such that $AB = 4$ and $(\overrightarrow{AB}\ ;\ \overrightarrow{AC}) = \dfrac{\pi}{3}\ \ (2\pi)$



Let I and E be the respective mid points of $[BC]$ and $[AC]$.

1- Let S be the similitude such that $S(A) = C$ and $S(C) = B$.
Determine the ratio and an angle of S .

2- a) Justify the existence of a rotation R that transforms A into E and B into C .
b) Determine an angle of R and construct , <u>with justification</u> , its center O .

3- a) Determine $R \circ S(C)$ and $S \circ R(B)$.
b) Determine the center , the ratio and an angle of each of the similitudes $R \circ S$ and $S \circ R$.

4- Let M and M' be the points defined by $\overrightarrow{AM} = k\,\overrightarrow{AB}$ and $\overrightarrow{EM'} = k\,\overrightarrow{EC}$ where k is a real number .
Prove that $M' = R(M)$. Deduce the nature of triangle OMM' .

5- Let (Γ) be the circle of center N circumscribed about the triangle OMM' .
a) Prove that the point A belongs to (Γ) .

b) Prove that N is the image of M by a similitude f of center O whose ratio and angle are to be determined .

c) Prove that $f(B)=E$ and determine $f(A)$

d) Determine the set (δ) of N as k traces IR . Draw (δ) .

Exercise 3 **(8 points)**

The complex plane is referred to a direct orthonormal system $(O;\vec{u},\vec{v})$.
To each point M of affix z , we associate the points M' and M'' of respective affixes z' and z'' such that $z'=z-2$ and $z''=z^2-z$.

1- Let $z=x+iy$ where x and y are two real numbers .

a) Determine the algebraic form of each of the complex numbers z' and z'' in terms x and y .

b) Find the points M for which the two points M' and M'' belong to the axis of ordinates .

2- a) Verify that $z'\neq z$ and determine the algebraic form of the number $\dfrac{z''-z}{z'-z}$ in terms of x and y .

b) Deduce the set of points M of plane for which the points M , M' and M'' are collinear .

c) Determine the set of points M that belong to the circle of diameter $[M'M'']$.

3- Let (H) be the curve of equation $x^2-y^2-2x=0$.

a) Prove that (H) is a rectangular hyperbola whose center , vertices and the equations of the asymptotes are to be determined .

b) $P(\alpha\,;\beta)$ is a point of (H) such that $(\vec{u},\overrightarrow{OP})=\theta\ (2\pi)$ where $0\le\theta<\dfrac{\pi}{4}$.

Prove that $OP=\dfrac{2\cos\theta}{\cos 2\theta}$. Determine θ , α and β when $OP=2\sqrt{3}$.

Exercise 4 **(7 points)**

An urn contains three perfect dice ; two of them are blue and each has six faces numbered 1 through 6 while the third one is red and has two faces numbered 1 and four faces numbered 6 .

1- We draw at random one die from the urn and we throw it . Consider the following events :
B : " the drawn die is blue " ; R : " the drawn die is red " and S : " the thrown die shows a 6 ".
Prove that $p(S)=\dfrac{1}{3}$.

2- In this part , we draw at random one die from the urn , then we throw this die n consecutive times .
Denote by S_n the event : " we get a 6 at each of the n throws " .

a) Calculate the probability of the event " the drawn die is blue and we get a 6 at each of the n throws " .

b) Prove that $p(S_n)=\frac{2}{3}\left(\frac{1}{6}\right)^n+\frac{1}{3}\left(\frac{2}{3}\right)^n$ and justify the value of $p(S)$ obtained in 1.

c) For all natural numbers $n \geq 1$, denote p_n the probability of having the red die knowing that we got a 6 at each of the n throws . Prove that $p_n=\dfrac{1}{2\left(\frac{1}{4}\right)^n+1}$.

d) Determine the least natural number $n \geq 1$ such that $p_n \geq 0.999$.

Exercise 5 **(7 points)**

Consider the function f defined on $]0;+\infty[$ by $f(x)=\dfrac{1+2\ell n\,x}{x^2}$.

1- The table of variations below is that of the of function g defined on $[2\ ;+\infty[$ by $g(x)=x-1-2\ell n\,x$.

x	2		$+\infty$
$g'(x)$	0	$+$	
$g(x)$	$1-\ell n4$	$\nearrow$	$+\infty$

a) Calculate $g(3)$ and $g(4)$ then prove that the equation $g(x)=0$ has exactly one root α belonging to the interval $]3\,;\,4[$.

b) Calculate $f(x)-\frac{1}{x}$ in terms of $g(x)$ and prove that , for all $x \geq 4$, $0<f(x)<\frac{1}{x}$. (1)

2- Consider the sequence (I_n) defined , for $n \geq 1$, by $I_n=\displaystyle\int_n^{n+1} f(x)\,dx$.

a) Using the inequality (1), prove that , for all natural numbers $n \geq 4$, $0<I_n<\ell n\left(\frac{n+1}{n}\right)$.

b) Deduce the limit of the sequence (I_n) .

3- Let $S_n=I_1+I_2+I_3+\cdots\cdots+I_n$.

a) Using integration by parts , prove that $\displaystyle\int f(x)\,dx=-\frac{3+2\ell n\,x}{x}+C$. (C is a constant)

b) Calculate S_n in terms of n.

c) Determine the limit of S_n as n tends to $+\infty$.

Exercise 6 **(12 points)**

A- Consider the function f defined on the set IR of real numbers by $f(x)=e^{2x}(e^x-2)^2$.

1- a) Prove that , for all real numbers x , $f'(x)=4e^{2x}(e^x-1)(e^x-2)$.

b) Set up the table of variations of f .

2- Draw the representative curve (C) of f in an orthonormal system $(O\ ;\ \vec{i}\ ,\ \vec{j}\ )$. **(*Unit* : 3 *cm*)** .

B- Consider the function F defined on IR by $F(x)=\dfrac{1}{12}(e^x-2)^3(3e^x+2)$.

1- Determine the sign of $F(x)$.

2- a) Prove that F is the antiderivative (primitive) of f on IR such that $F(\ell n2)=0$.

b) Set up the table of variations of F .

3- Let (γ) be the representative curve of F in the same orthonormal system as (C).

a) Determine the position of (γ) with respect to (C) and prove that these two curves are tangent at a point to be determined.

b) Prove that the straight line of equation $y=-\dfrac{4}{3}$ is asymptote to (γ) at $-\infty$.

c) Draw (γ) in the same system as (C).

4- Let m be a real number such that $m\leq 0$. Consider the domain (D) bounded by (C) , the axis of abscissas , and the straight lines of equations $x=m$ and $x=\ell n2$.

a) Calculate the measure $S(m)$, in *units of area* , of the area of the domain (D) in terms of $F(m)$ and determine $\lim\limits_{m\to-\infty} S(m)$.

b) Prove that , for all $m\in]-\infty\ ;0]$, $\dfrac{5}{12}\leq S(m)<\dfrac{4}{3}$.

c) a being a given number such that $\dfrac{5}{12}\leq a<\dfrac{4}{3}$, describe the construction that allows to determine graphically the real number m such that $S(m)=a$.

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SOLUTION

Exercise 1 (points)

1- a) The equation $x^3-2x^2+xy^2+2y^2=0$ of (γ) is equivalent to $x^2(x-2)+y^2(x+2)=0$;
that is $y^2(2+x)=x^2(2-x)$ (1).
For $x=-2$, (1) becomes $0=16$ which is impossible .
For $x\neq -2$, (1) is equivalent to $y^2=\left(\dfrac{2-x}{2+x}\right)x^2$, which is defined for all real numbers x such that $\dfrac{2-x}{2+x}\geq 0$; that is $-2<x\leq 2$.
Finally , the set of abscissas of the points of (γ) is the interval $I=]-2\ ;\ 2]$.

b) An equation of the symmetric of (γ) with respect to the axis of abscissas is
$x^2(x-2)+(-y)^2(x+2)=0$ which is $x^2(x-2)+y^2(x+2)=0$, that of (γ).
Therefore , the axis of abscissas is an axis of symmetry of (γ) .

2- a) By symmetry with respect to the axis of abscissas, the required volume V is equal
$\pi\int_0^2 y^2dx$ *units of volume* ;

$$\int_0^2 y^2\,dx=\int_0^2\frac{-x^3+2x^2}{x+2}\,dx=\int_0^2\left(-x^2+4x-8+\frac{16}{x+2}\right)dx=\left[-\frac{x^3}{3}+2x^2-8x+16\ell n(x+2)\right]_0^2.$$

$$=16(\ell n2-\frac{2}{3})\ ;\ \text{therefore}\ \ V=16(\ell n2-\frac{2}{3})\pi\ \textit{units of volume}\ .$$

b) If $\left\|\vec{i}\right\|=\left\|\vec{j}\right\|=2cm$, then 1 *unit of volume* = $8cm^3$;

Therefore $V=16\pi(\ell n2-\frac{2}{3})\pi\times 8=128(\ell n2-\frac{2}{3})\pi\ \ cm^3$.

Exercise 2 **(points)**

The triangle ABC is semi equilateral having $AB = 4$, $(\overrightarrow{AB}\ ;\ \overrightarrow{AC}) = \frac{\pi}{3}\ (2\pi)$ and $(\overrightarrow{BC}\ ;\ \overrightarrow{BA}) = \frac{\pi}{2}\ (2\pi)$ then $AC = 2AB = 8$ and $BC = \sqrt{3}\,AB = 4\sqrt{3}$.

I and E are the respective mid points of $[BC]$ and $[AC]$.

1- $S(A) = C$ and $S(C) = B$ where $BC = \frac{\sqrt{3}}{2}AC$ and $(\overrightarrow{AC}\ ;\ \overrightarrow{CB}) = -\frac{5\pi}{6}\ (2\pi)$, then $\frac{\sqrt{3}}{2}$ is the ratio of S and $-\frac{5\pi}{6}$ is an angle of S .

2- a) $AB = EC = 4$ and $\overrightarrow{AB} \neq \overrightarrow{EC}$, then there exists a rotation R that transforms A into E and B into C .

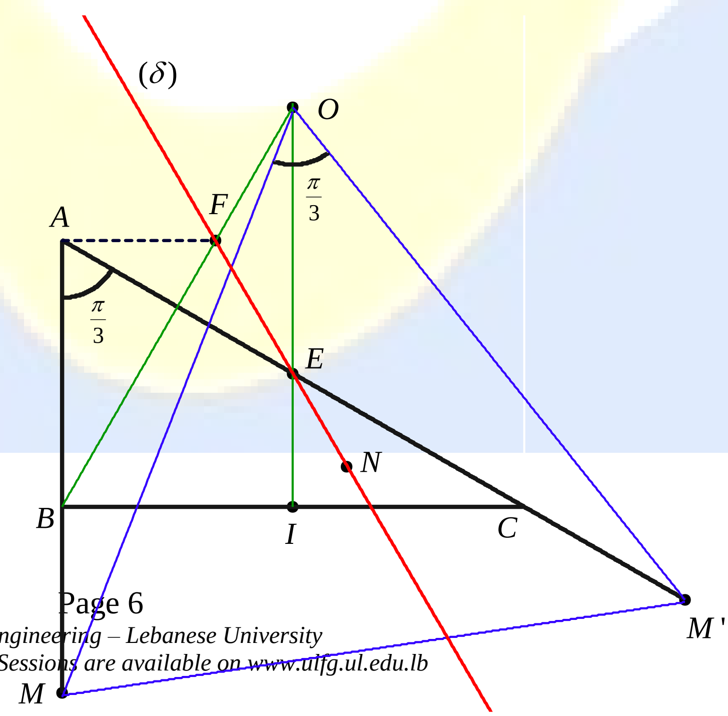
b) $(\overrightarrow{AB}\ ;\ \overrightarrow{EC}) = (\overrightarrow{AB}\ ;\ \overrightarrow{AC}) = \frac{\pi}{3}\ (2\pi)$, then $\frac{\pi}{3}$ is an angle of R ; its center is the point O such that $OA = OE$ and $OB = OC$; it is the point of intersection O of the perpendicular bisector of $[AE]$ that passes through B (ABE is equilateral) and the perpendicular bisector of $[BC]$ that passes through E (BEC is isosceles at E).

3- a) $R \circ S(C) = R(S(C)) = R(B) = C$ and $S \circ R(B) = S(R(B)) = S(C) = B$.

b) $R \circ S = S(O\ ;1\ ;\frac{\pi}{3}) \circ S(.....\ ;\ \frac{\sqrt{3}}{2}\ ;-\frac{5\pi}{6}) = S(B\ ;\ \frac{\sqrt{3}}{2}\ ;-\frac{5\pi}{6}+\frac{\pi}{3} = -\frac{\pi}{2})$ and $S \circ R = S(C\ ;\ \frac{\sqrt{3}}{2}\ ;-\frac{\pi}{2})$

4- $\overrightarrow{AM} = k\,\overrightarrow{AB}$ and $\overrightarrow{EM'} = k\,\overrightarrow{EC}$ where $R(A) = E$ and $R(B) = C$, then $M' = R(M)$.

$M' = R(M)$ where $R = r(O\ ;\ \frac{\pi}{3})$, then $OM = OM'$ and $(\overrightarrow{OM}\ ;\ \overrightarrow{OM'}) = \frac{\pi}{3}\ (2\pi)$, then OMM' is a direct equilateral triangle .



5- (γ) is the circle of center N circumscribed about the triangle OMM' .

a) $(\overrightarrow{AM}\ ;\ \overrightarrow{AM'}) = (\overrightarrow{AB}\ ;\ \overrightarrow{AC}) = \frac{\pi}{3}\ (2\pi)$, then $(\overrightarrow{AM}\ ;\ \overrightarrow{AM'}) = (\overrightarrow{OM}\ ;\ \overrightarrow{OM'})$ and A , O , M , M' are cyclic ; therefore the point A belongs to (γ) .

b) OMM' is a direct equilateral triangle , then N is its center of gravity ; therefore $(\overrightarrow{OM}\ ;\ \overrightarrow{ON}) = \frac{\pi}{6}\ (2\pi)$ and $ON = \frac{\sqrt{3}}{3}\ OM$; therefore N is the image of M by the similitude f of center O , ratio $\frac{\sqrt{3}}{3}$ and angle $\frac{\pi}{6}$.

c) $R(B) = C$, then OBC is a direct equilateral triangle .

E is on the perpendicular bisector (OI) of $[BC]$ and $EI = \frac{1}{2}AB = \frac{1}{2}EB$, then E is the center of the circle circumscribed about OBC ; therefore $f(B) = E$.

$R(A) = E$, then OAE is a direct equilateral triangle ; therefore $f(A)$ is F , the center of the circle circumscribed about OAE .

d) As k traces IR , the point M , such that $\overrightarrow{AM} = k\,\overrightarrow{AB}$, traces the straight line (AB) and its image N by f traces the straight line $f\big((AB)\big)$ which is (EF) since $f(A) = F$ and $f(B) = E$.

Therefore , the set (δ) of N as k traces IR is the straight line $(\delta) = (EF)$.

Drawing (δ) .

Exercise 3 **(points)**

1- a) $z' = z - 2 = x - 2 + iy$ and $z'' = z^2 - z = x^2 - y^2 + 2xyi - x - iy = x^2 - y^2 - x + y(2x-1)i$.

b) M' and M'' belong to $y'y$ if and only if $\text{Re}(z') = \text{Re}(z'') = 0$; that is $x = 2$ and $x^2 - y^2 - x = 0$; $x = 2$ and $y^2 = 2$; therefore , M is one of the points with affixes $2 + \sqrt{2}i$ and $2 - \sqrt{2}i$.

2- a) $z' = z - 2 \neq z$ and $\dfrac{z''-z}{z'-z} = \dfrac{z^2 - 2z}{z'-z} = \dfrac{x^2 - y^2 + 2xyi - 2x - 2yi}{-2} = \dfrac{x^2 - y^2 - 2x}{-2} + y(1-x)i$.

b) $M' \neq M$, then the points M , M' and M'' are collinear if and only if $\dfrac{z''-z}{z'-z}$ is real ; that is $(x-1)y = 0$; $x = 1$ or $y = 0$; therefore , the set of M is the union of the axis of abscissas and the straight line of equation $x = 1$.

c) M belongs to the circle of diameter $[M'M'']$ if and only if $\dfrac{z''-z}{z'-z} = 0$ or $\dfrac{z''-z}{z'-z}$ is pure imaginary ;

that is $\text{Re}\left(\dfrac{z''-z}{z'-z}\right) = 0$. Therefore , the set of M is the curve of equation $x^2 - y^2 - 2x = 0$.

3- (H) is the curve of equation $x^2 - y^2 - 2x = 0$.

a) An equation of (H) is $(x-1)^2 - y^2 = 1$, then (H) is a rectangular hyperbola of center $I(1\,;0)$.
The focal axis is the axis of abscissas and $a = 1$, then the vertices of (H) are O and $A(2\,;0)$.
The asymptotes of (H) are the straight lines of equations $y = x - 1$ and $y = -x + 1$.

b) $(\vec{u}\ ,\ \overrightarrow{OP}) = \theta \quad (2\pi)$ then , the coordinates of P are $\alpha = OP\cos\theta$ and $\beta = OP\sin\theta$.
P is a point of (H) then , $\alpha^2 - 2\alpha - \beta^2 = 0$; that is $OP^2\cos^2\theta - 2OP\cos\theta - OP^2\sin^2\theta = 0$;
$OP\cos 2\theta = 2\cos\theta$ where $\cos 2\theta \neq 0$ since $0 \leq 2\theta < \dfrac{\pi}{2}$; therefore $OP = \dfrac{2\cos\theta}{\cos 2\theta}$.

$OP = 2\sqrt{3}$ if and only if $\dfrac{\cos\theta}{\cos 2\theta} = \sqrt{3}$; $\sqrt{3}(2\cos^2\theta - 1) = \cos\theta$; $2\sqrt{3}\cos^2\theta - \cos\theta - \sqrt{3} = 0$;

Therefore , $\cos\theta = \dfrac{\sqrt{3}}{2}$ $(0 \leq \theta < \dfrac{\pi}{4})$; $\theta = \dfrac{\pi}{6}$ *rad* .

When $\theta = \dfrac{\pi}{6}$ *rad* , $\alpha = OP\cos\theta = 3$ and $\beta = OP\sin\theta = \sqrt{3}$.

Exercise 4 **(points)**

1- When we draw at random one die from the urn that contains 2 blue dice and one red , $p(B)=\frac{2}{3}$ and $p(R)=\frac{1}{3}$.

If a blue die is drawn and tossed then , the probability of getting a 6 is $p(S/B)=\frac{1}{6}$.

If the red die is drawn and tossed then , the probability of getting a 6 is $p(S/R)=\frac{4}{6}=\frac{2}{3}$.

Therefore , $p(S)=p(S\cap B)+p(S\cap R)=p(B)\times p(S/B)+p(R)\times p(S/R)=\frac{2}{3}\times\frac{1}{6}+\frac{1}{3}\times\frac{2}{3}=\frac{1}{3}$.

2- a) The required probability is $p(S_n\cap B)=p(B)\times p(S_n/B)=\frac{2}{3}\times\left(\frac{1}{6}\right)^n$

b) $p(S_n)=p(S_n\cap B)+p(S_n\cap R)=p(B)\times p(S_n/B)+p(R)\times p(S_n/R)=\frac{2}{3}\times\left(\frac{1}{6}\right)^n+\frac{1}{3}\left(\frac{2}{3}\right)^n$.

For $n=1,\ p(S)=p(S_1)=\frac{2}{3}\times\left(\frac{1}{6}\right)+\frac{1}{3}\left(\frac{2}{3}\right)=\frac{1}{3}$.

c) $p_n=p(R/S_n)=\dfrac{p(R\cap S_n)}{p(S_n)}=\dfrac{\frac{1}{3}\left(\frac{2}{3}\right)^n}{\frac{2}{3}\times\left(\frac{1}{6}\right)^n+\frac{1}{3}\left(\frac{2}{3}\right)^n}=\dfrac{\frac{1}{3}\left(\frac{2}{3}\right)^n}{\frac{2}{3}\times\left(\frac{2}{3}\times\frac{1}{4}\right)^n+\frac{1}{3}\left(\frac{2}{3}\right)^n}=\dfrac{1}{2\left(\frac{1}{4}\right)^n+1}$.

d) $p_n\geq 0.999$ is equivalent to $1000\geq 1998\left(\frac{1}{4}\right)^n+999$; $1\geq 1998\left(\frac{1}{4}\right)^n$; $4^n\geq 1998$; $n\geq\frac{\ell n1998}{\ell n4}\approx 5.48$.

Therefore , the least natural number $n\geq 1$ such that $p_n\geq 0.999$ is 6 .

Exercise 5 **(points)**

1- $g(x) = x - 1 - 2\ell n x$.

a) $g(3) = 2 - 2\ell n 3$ and $g(4) = 3 - 2\ell n 4$ then $g(3) = 2 - 2\ell n 3 < 0 < g(4) = 3 - 2\ell n 4$

g is continuous and strictly increasing in $[2\,;+\infty[$, then the equation $g(x) = 0$ has a unique root α such that $3 < \alpha < 4$.

b) $f(x) - \dfrac{1}{x} = \dfrac{-x + 1 + 2\ell n x}{x^2} = -\dfrac{g(x)}{x^2}$.

For all x in $[4;+\infty[$, $\ell n x > 0$, then $f(x) > 0$.

For all x in $[4;+\infty[$, $g(x) > g(\alpha) = 0$, then $f(x) - \dfrac{1}{x} < 0$; $f(x) < \dfrac{1}{x}$.

Finally , for all x in $[4;+\infty[$, $0 < f(x) < \dfrac{1}{x}$. (1)

2- The sequence (I_n) is defined , for $n \geq 1$, by $I_n = \displaystyle\int_n^{n+1} f(x)\,dx$.

a) For all x in $[4;+\infty[$, $0 < f(x) < \dfrac{1}{x}$, then , for all natural numbers $n \geq 4$, $0 < I_n < \displaystyle\int_n^{n+1} \frac{dx}{x}$ where

$\displaystyle\int_n^{n+1} \frac{dx}{x} = \big[\ell n|x|\big]_n^{n+1} = \ell n(n+1) - \ell n\, n = \ell n\left(\frac{n+1}{n}\right)$; therefore $0 < I_n < \ell n\left(\dfrac{n+1}{n}\right)$

b) $\displaystyle\lim_{n\to+\infty} \ell n\left(\frac{n+1}{n}\right) = \ell n\,1 = 0$ and $0 < I_n < \ell n\left(\dfrac{n+1}{n}\right)$, then the limit of the sequence (I_n) is 0.

3- a) Let $u = 1 + 2\ell n x$ and $v' = \dfrac{1}{x^2}$, then $u' = \dfrac{2}{x}$ and $v = \dfrac{-1}{x}$.

Using integration by part , $\displaystyle\int f(x)\,dx = -\frac{1+2\ell n x}{x} + 2\int \frac{1}{x^2}\,dx = -\frac{1+2\ell n x}{x} - \frac{2}{x} = -\frac{3+2\ell n x}{x}$.

b) $S_n = I_1 + I_2 + I_3 + \cdots\cdots + I_n = \displaystyle\int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx + \cdots\cdots + \int_{n-1}^{n} f(x)\,dx + \int_n^{n+1} f(x)\,dx$;

$S_n = \displaystyle\int_1^{n+1} f(x)\,dx = \left[\frac{3+2\ell n x}{x}\right]_{n+1}^{1} = 3 - \frac{3}{n+1} - 2\frac{\ell n(n+1)}{n+1}$.

c) $\lim\limits_{n\to+\infty}\dfrac{3}{n+1}=\lim\limits_{n\to+\infty}\dfrac{\ln(n+1)}{n+1}=0$, then $\lim\limits_{n\to+\infty}S_n=3$.

Exercise 6 **(points)**

A- 1- a) $f(x)=e^{2x}(e^x-2)^2$, then

$$f'(x)=2e^{2x}(e^x-2)^2+2e^{2x}(e^x-2)(e^x)=2e^{2x}(e^x-2)(e^x-2+e^x)=4e^{2x}(e^x-1)(e^x-2) .$$

b) $\lim\limits_{x\to-\infty}e^{2x}=0$ then , $\lim\limits_{x\to-\infty}f(x)=0$;

$\lim\limits_{x\to+\infty}e^x=\lim\limits_{x\to+\infty}e^{2x}=+\infty$ then , $\lim\limits_{x\to+\infty}f(x)=+\infty$.

The sign of $f'(x)$ is that of $(e^x-1)(e^x-2)$;

it changes at $x=0$ and at $x=\ln 2$.

Table of variations of f

x	$-\infty$		0		$\ln 2$		$+\infty$
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$	0	↗	1	↘	0	↗	$+\infty$

Figure 52

2- $\lim\limits_{x\to-\infty}f(x)=0$ then , the axis of abscissas is asymptote to (C) at $-\infty$.

$\lim\limits_{x\to+\infty}f(x)=+\infty$ and $\lim\limits_{x\to+\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to+\infty}\dfrac{e^x}{x}e^x(e^x-2)^2=+\infty$ since $\lim\limits_{x\to+\infty}\dfrac{e^x}{x}=+\infty$ then , (C) has at $+\infty$ an asymptotic direction parallel to the axis of ordinates .

Drawing (C) . **(*Unit* : 3 *cm*) .**

B- 1- $F(x)=\dfrac{1}{12}(e^x-2)^3(3e^x+2)$, then $F(x)$ has the sign of e^x-2.

$F(\ln 2)=0$; if $x<\ln 2$, $F(x)<0$ and if $x>\ln 2$, $F(x)>0$.

2- a) $F'(x)=\dfrac{1}{4}(e^x-2)^2(e^x)(3e^x+2)+\dfrac{1}{12}(e^x-2)^3(3e^x)=\dfrac{1}{12}(e^x-2)^2(9e^{2x}+6e^x+3e^{2x}-6e^x)$

$=e^{2x}(e^x-2)^2=f(x)$, then F is the antiderivative of f on IR such that $F(\ln 2)=0$.

b) $\lim\limits_{x\to-\infty}F(x)=-\dfrac{4}{3}$ and $\lim\limits_{x\to+\infty}F(x)=+\infty$

$F'(x)=e^{2x}(e^x-2)^2\geq 0$

table of variations of F .

x	$-\infty$		$\ln 2$		$+\infty$
$F'(x)$		$+$	0	$+$	
$F(x)$	$-\frac{4}{3}$	↗	0	↗	$+\infty$

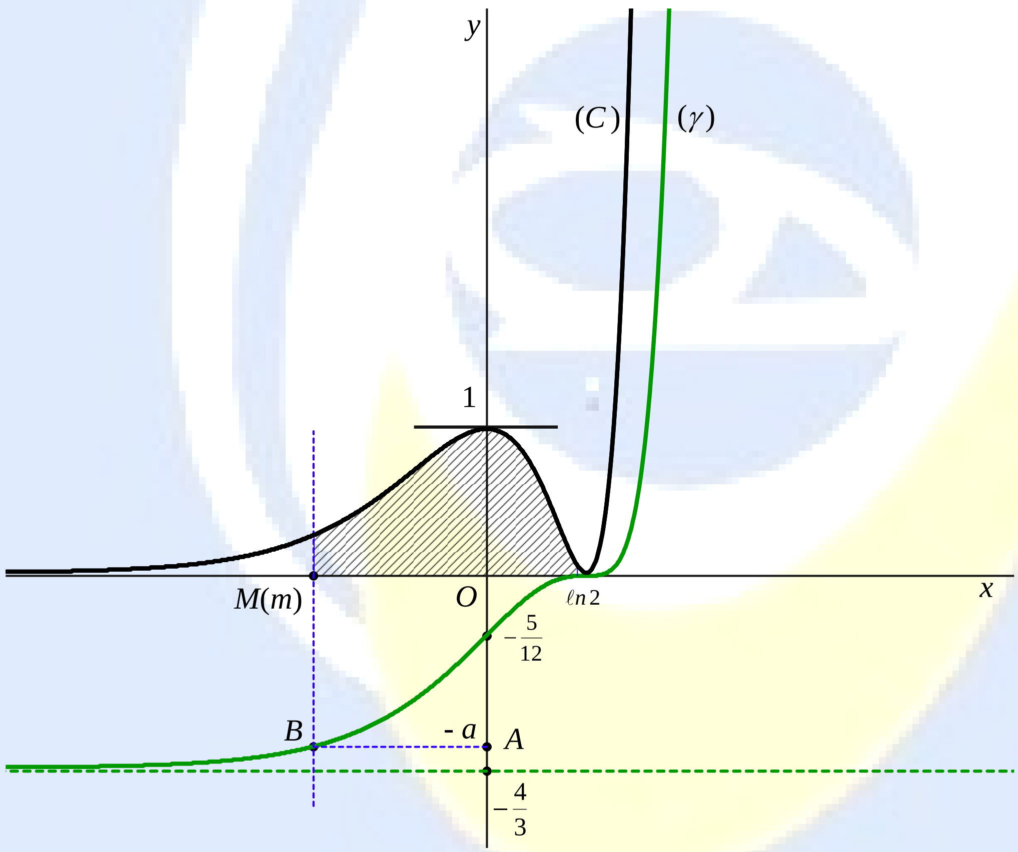
3- a) $f(x)-F(x)=e^{2x}(e^x-2)^2-\frac{1}{12}(e^x-2)^3(3e^x+2)=\frac{1}{12}(e^x-2)^2(12e^{2x}-3e^{2x}+4e^x+4)$

$f(x)-F(x)=\frac{1}{12}(e^x-2)^2(9e^{2x}+4e^x+4)\geq 0$, then (C) and (γ) have one common point $T(\ell n2\,;\,0)$ and , for all $x\neq \ell n2$, (C) lies above (γ) .

(C) and (γ) are tangent at T since $F'(\ell n2)=f'(\ell n2)$ (both are 0) .

b) $\lim\limits_{x\to-\infty} F(x)=-\frac{4}{3}$, then the straight line of equation $y=-\frac{4}{3}$ is asymptote to (γ) at $-\infty$.

c) Drawing (γ) .



4- Let m be a real number such that $m\leq 0$.

a) f is a positive function then , $S(m) = \int_{m}^{\ell n 2} f(x)\, dx = -F(m)$.

$\underset{m \to -\infty}{\ell im} S(m) = -\underset{m \to -\infty}{\ell im} F(m) = \frac{4}{3}$.

b) F being strictly increasing , then For all $m \in]-\infty\, ;\, 0]$, $\underset{m \to -\infty}{\ell im} F(m) < F(m) \leq F(0)$; that is

$-\frac{4}{3} < F(m) \leq -\frac{5}{12}$, then $\frac{5}{12} \leq S(m) < \frac{4}{3}$.

c) a being a given number such that $\frac{5}{12} \leq a < \frac{4}{3}$, take the point A of ordinate $-a$ on the axis of ordinates ; the parallel to the axis of abscissas drawn through A cuts (γ) at a point B , then the parallel to the axis of ordinates drawn through B cuts the axis of abscissas at the point M such that $m = (abscissa\, of\;\; M) = -\, OM$.