LEBANESE UNIVERSITY

FACULTY OF ENGINEERING

الجامعة اللبنانية
كلية الهندسة

Entrance exam 2019-2020 **Physics (Bac. Lebanese)** **July 2019**
Duration 2 h

Exercise I: Polonium 210: The murderer which devours red blood cells [21 points]

"Polonium 210 ($^{210}_{84}Po$) is a million times more toxic than cyanide; one hundredth of a milligram (10 micrograms) is enough to kill, in a few weeks, a man of "middleweight"…
It must also be obtained quickly, because it loses half of its radioactivity every 138 days; it must have been recently made by irradiation of bismuth $^{209}_{83}Bi$ by capturing a neutron in a nuclear reactor...
The poison, after ingestion, passes from the stomach into the blood circulation. Each atom of polonium 210 is then carrying an alpha projectile ejected at high speed, enough to literally burn all the cells of the body, red blood cells first, and cause a so-called "multifactorial" death...
According to Fabien Gruhier- Nouvel Observateur 11-17 January 2007

Data: Masses of some nuclei or particles: m($^{209}_{83}Bi$) = 208.934860 u; m($^{210}_{83}Bi$) = 209.938584 u; m($^{210}_{84}Po$) = 209.936800 u; m($^{206}_{82}Pb$) = 205.929489 u; m($^{4}_{2}He$) = 4.00151 u; m($^{1}_{1}H$) = 1.007276 u ; m($^{1}_{0}n$) = 1.008665 u ; m($^{0}_{-1}e$) = 5.49×10^{-4} u ;
Atomic molar mass: M($^{210}_{84}Po$) = 210 g mol^{-1} ; Avogadro's number : $N_A = 6.022 \times 10^{23}$ mol^{-1} ;
1 u= 931.5 MeV/c^2 = 1.66054×10^{-27} kg ; c = 2.99792×10^{8} m s^{-1} ; h = 6.626×10^{-34} J·s ; 1 eV = 1.602×10^{-19} J.

Part A: Obtaining polonium

1 The bismuth 209 captures in a first step a neutron; $^{209}_{83}Bi + ^{1}_{0}n \rightarrow ^{210}_{83}Bi$; the resulting nucleus undergoes disintegration and the daughter nucleus obtained is a polonium 210 nucleus. Write, with justification, the equation of this disintegration (the daughter nucleus will not be written in an excited state).

2 .1 Calculate the binding energy per nucleon of the bismuth 210 nucleus.
2 .2 Knowing that the binding energy per nucleon of the bismuth 209 nucleus is 7.839 MeV/nucleon, deduce the nucleus that is more stable than the other.

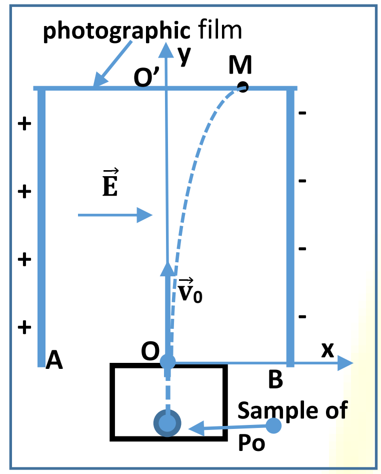
Part B: Polonium 210 disintegration

A single gram of polonium 210 has an activity of 166 761 billion becquerels and therefore emits 166 761 billion alpha particles per second.

1. Write the equation of decay of a $^{210}_{84}Po$ nucleus by specifying the conservation laws used (it is assumed that the daughter nucleus is formed in the ground state).

2. Calculate, in MeV, the energy released by the decay of a polonium 210 nucleus.

3. The parent nucleus is initially at rest and the released energy appears as a kinetic energy for the particle α and another one for the daughter nucleus. Determine the kinetic energy KE(α) of a particle α, assuming that during this disintegration, we have the conservation of the linear momentum of the system (daughter nucleus, α).



Doc. 1

4. To verify the value of the KE(α), a beam of these particles is deflected by a uniform electric field $\vec{E}$ (Doc. 1).
At the instant $t_0 = 0$, a mono-kinetic energy beam of these α particles penetrates at point O, located at the same distance from the two parallel and horizontal plates of a plane capacitor, between which exists the electric field $\vec{E}$ of magnitude

$E = 5\times10^6$ V/m. Each particle is moving with a velocity $\vec{v}_0 = v_0\,\vec{j}$, the linear momentum being: $\overrightarrow{P_0} = P_{0x}\,\vec{i} + P_{0y}\vec{j} = m\overrightarrow{v_0}$.
At an instant t, each particle has a linear momentum $\vec{P} = P_x\,\vec{i} + P_y\vec{j}$ and undergoes the action of the electric force $\vec{F} = q_\alpha\,\vec{E} = q_\alpha E\,\vec{i}$. At the distance
L = OO' = 10 cm is placed a photographic film on which the impacts of the alpha particles create bright spots. (In the absence of $\vec{E}$, a spot is formed at O' of the film). The particles are deflected towards the plate B and a spot is formed a point M of the film at a distance d = 4.713 mm from point O'.
The weight of the particle α is neglected and this experiment is carried out in vacuum.

4.1. By applying Newton's second law, show that, at an instant t, the frame of reference being $(O, \vec{i}, \vec{j})$: P_y remains constant and equal to P_{0y} and P_x is of the form $P_x = at + b$, specifying the expression of a and the value of b.
4.2. Show that, at an instant t, $x = \frac{eE}{m}t^2$ and $y = v_0\,t$, x and y being the coordinates of the particle.
4.3. Deduce the equation of its trajectory.

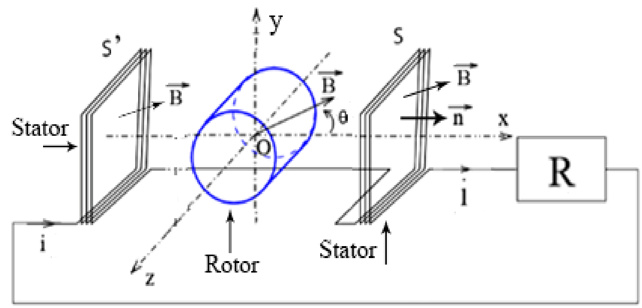
4.4. Determine, in eV, its kinetic energy at O.

5. The experiment shows that some particles have kinetic energy $KE_1(\alpha) = 5.30$ MeV and others have kinetic energy $KE_2(\alpha) = 4.50$ MeV, and that some disintegrations are accompanied by the emission of a γ radiation. Interpret the existence of this γ radiation and calculate its wavelength.

6. Justify, by calculation, the "sentence" a single gram of polonium 210 has an activity of 166 761billions of becquerels".

Exercise II Electromechanical energy converter: prototype [18 points]

An alternator allows to transform mechanical energy into electric energy. Its functioning is based on the phenomenon of the electromagnetic induction. It consists of a rotor and a stator. The rotor, rotating around its Oz axis, can be seen as a magnet, creating in its vicinity a magnetic field $\vec{B}$, supposed of constant magnitude, the lines of field $\vec{B}$ being parallel to the plane (xOy). Each vector $\vec{B}$ is a rotating vector, located, at an instant t, by its angular abscissa θ with respect to the Ox axis. The normal $\vec{n}$ to the coils (S) and (S') is parallel to the Ox axis and have the same direction (Doc1).



Doc. 1

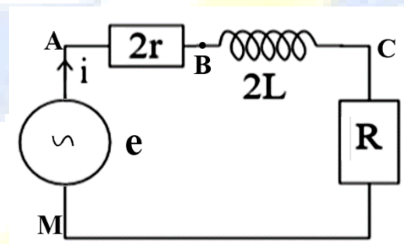
The rotor is driven by a mechanical system that exerts a motive torque whose moment is written as $\vec{M}_m = M_m\,\vec{k}$. where $M_m = 3.8\times10^{-2}$ m·N. Finally, we take the resistive torque of the forces of friction $\vec{M}_f = -M_f\,\vec{k}$ where $M_f > 0$. The stator consists of a combination of two coils (S) and (S') and a resistor (R) of resistance R = 2.0 Ω; each of the two coils, of resistance r = 1,0 Ω and of inductance L = 0.20 mH, is made up of N rectangular loops, each of cross sectional area A. It is considered that the rotor angular velocity ω is a positive constant with θ = ω t.

1. Determine, at an instant t, the total flux Φ through both coils (S) and (S').

2. Deduce that the induced electromotive force "e" created has the expression:

$e = D\,\omega\sin(\omega t)$ with D = 2NAB.

3. The electric circuit equivalent to the circuit mentioned above is in agreement with that of Doc 2.



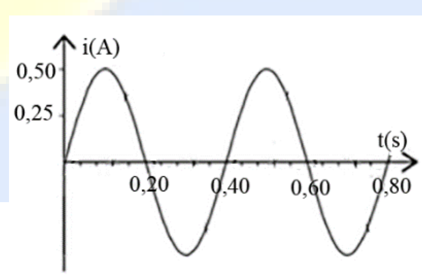
Doc. 2

In steady state, the circuit carries a current $i = I_m \sin(\omega t - \varphi)$ where $-\frac{\pi}{2}$ rad $< \varphi <$ $\frac{\pi}{2}$ rad.

3.1. By applying the law of addition of voltages and giving t two particular values, show that:

$i = \frac{D\cdot\omega}{\sqrt{(2r+R)^2+(2L\omega)^2}}\sin(\omega t - \varphi)$ with $\tan\varphi = \frac{2L\omega}{2r+R}$ and $\cos\varphi = \frac{2r+R}{\sqrt{(2r+R)^2+(2L\omega)^2}}$.

3.2. The curve of the document 3 is that of the current i as a function of time.



3.2.1. Determine the value of the angular velocity ω of the rotor.

3.2.2. Write, in terms of the data, the expression of the average power P delivered by the coils (S) and (S') to (R) and calculate its value.

4.1. Determine the mechanical power P_m received by the alternator.

4.2. Deduce the efficiency of the alternator.

4.3. Determine the value of M_f.

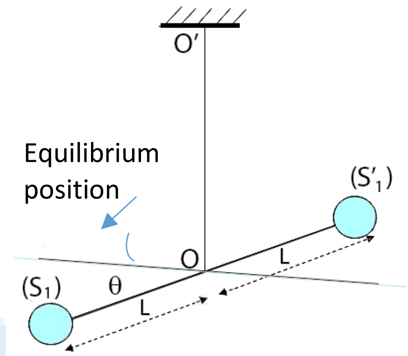
5. Determine the value of D.

Exercise III: Torsion pendulum [21 points]

A- Free modes

The torsion pendulum allows to determine some physical quantities.

A torsion pendulum consists of a vertical wire, of torsion constant C, to which is attached a set (Σ) consisting of a thin steel rod, of mass $m_T = 0.090$ kg and of length $2L = 0.20$ m, bearing, at one end, a small ball (S_1) and at the other another small ball (S'_1) identical to (S_1); each of the balls is considered as a point mass of mass m = 0.050 kg. The moment of inertia I_0 of the rod with respect to the vertical axis (Δ) coinciding with the wire and passing through O, the center of inertia of the rod, is $I_0 = \frac{1}{3}m_T \cdot L^2$.



Doc. 1

We rotate (Σ) horizontally in the positive direction around the axis (Δ) by an angle $\theta_0 = 0.080$ rad with respect to its equilibrium position, and we release it from rest at the instant $t_0 = 0$. At an instant t, the angular abscissa of (Σ) is θ and its angular velocity is $\dot{\theta} = \frac{d\theta}{dt}$; the wire, being thus twisted by an angle θ, exerts then a restoring torsion torque of moment $M_1 = -C\theta$ and stores a torsion elastic potential energy $PE_e = ½ C \theta^2$. The horizontal plane passing through O is taken as the reference level for the gravitational potential energy. (Doc. 1)

1. Free undamped mode

1.1. Show that the moment of inertia I_Δ of (Σ) with respect to the axis (Δ) is: $I_\Delta = 1.30\times10^{-3}$ kg·m^2.

1.2. By applying the conservation of the mechanical energy of the system (pendulum, Earth), determine the differential equation in θ that describes the motion of the pendulum.

1.3. Deduce the expression of the proper angular frequency ω_0 of the pendulum as well as that of its proper period T_0 in terms of the data.

2. Free damped mode

In reality, by keeping the same initial conditions, (Σ) undergoes in addition to the restoring torsion torque of moment M_1, at an instant t, the action of the frictional forces torque of moment $M_2 = -2\lambda\omega_0\dot{\theta}$, where λ is a positive constant.

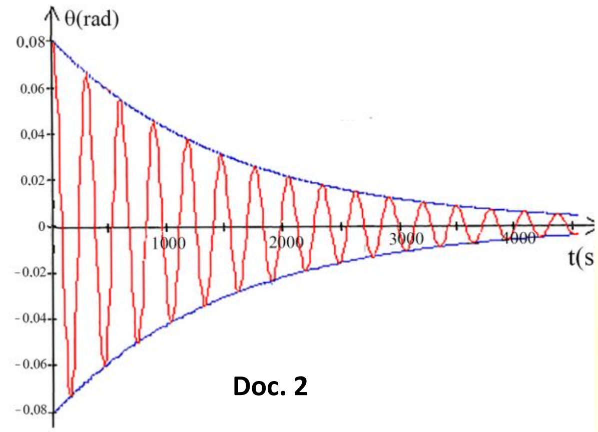
2.1. Show that the differential equation in θ that describes the pendulum's motion is written as:

$\ddot{\theta} + \frac{2\lambda\omega_0}{I_\Delta}\dot{\theta} + \frac{C}{I_\Delta}\theta = 0$

2.2. For $\lambda = 0.25$, the solution of the differential equation is written in the form: $\theta = A\,e^{-\lambda\omega_0 t}\cos(\omega t - \varphi)$, where A is a positive constant, $\omega = \omega_0\sqrt{1-\lambda^2}$ and $\tan\varphi = \frac{\lambda}{\sqrt{1-\lambda^2}}$ (Doc 2).

2.2.1. Indicate the mode that the pendulum undergoes and then specify whether the damping is strong or weak.

2.2.2. The document (Doc 2) shows the recording of the oscillations of the pendulum. Determine the value of ω and calculate the values of φ, C and A.

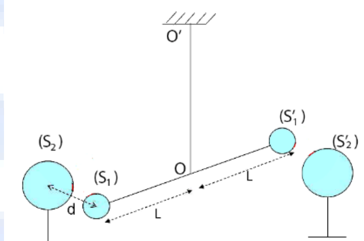


Doc. 2

B- Measurement of G

The magnitude F of the gravitational attractive force that is exerted between two point objects, of respective masses m_1 and m_2, separated by a distance r is written as: $F = G\frac{m_1 \cdot m_2}{r^2}$, where G is the universal gravitational constant.

(Σ) is maintained in its equilibrium position. We place a lead sphere of mass M = 30 kg, considered as a point mass, in front of each smallball and at a distance of d = 17.7 cm from each ball. We release (Σ) from rest at the instant $t_0 = 0$. At an instant t, the angular abscissa of (Σ) is θ (very small) and its angular velocity is $\dot{\theta} = \frac{d\theta}{dt}$ (Doc 3).(Take only the two interactions between the particles that are in front of each other).



Doc. 3

The pendulum performs oscillations, stabilizes and (Σ) takes a new equilibrium position (Doc 4).

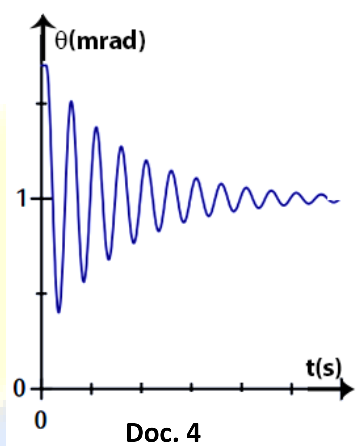
Take: sinθ ≈ θ and cos θ ≈ 1 (θ in rad).

1.1. At an instant t, during the oscillations, the distance r between the point masses (S_1) and (S_2) on one hand and (S'_1) and (S'_2) on the other hand is r = d - Lθ, with θ in rad. Justify.

1.2. Determine, at an instant t, during oscillations, the expressions and the values of the moments of the external forces and torques exerted on (Σ).

2. By applying the theorem of angular momentum, determine the differential equation in θ which describes the motion of (Σ).

3. Referring to Doc 4, deduce the value of G.



Doc. 4

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Answer of Physics

Exercise I: The murderer which devours red blood cells

Q		Note
A.1.	${}^{210}_{83}Bi \rightarrow {}^{210}_{84}Po + {}^{a}_{z}P$ According to the law of conservation of the mass number: 210 = 210 + a ⇒ a = 0. According to the law of conservation of the charge number: 83 = 84 + z ⇒ z = -1. Thus : ${}^{210}_{83}Bi \rightarrow {}^{210}_{84}Po + {}^{0}_{-1}e$	**1**
2.1.	The mass defect: $\Delta m = Z\, m_P + (A-Z)\, m_n - m({}^{210}_{83}Bi)$. $\Delta m = 83\times1.007276 + 127\times1.008665 - 209.938584 = 1.76578$ u. ¾ The binding energy of bismuth 210 is given by Einstein's relation: $E_\ell = \Delta m\cdot c^2 = 1.76578\times931.5 = 1644.8$ MeV ½ The binding energy per nucleon of bismuth 210 is: E_ℓ/nucleon (${}^{210}_{83}Bi$) =1644.8/210 = 7.832 MeV/nucleon. ½	**1.75**
2.2.	The bismuth nucleus 209 is more stable than the bismuth nucleus 210 because its binding energy per nucleon E_ℓ/nucleon(${}^{209}_{83}Bi$) = 7.839 MeV/nucleon is greater than that of the ${}^{210}Bi$ nucleus : E_ℓ/nucleon(${}^{210}_{83}Bi$) = 7.832 MeV/nucleon	**0.5**
B.1	${}^{210}_{84}Po \rightarrow {}^{A}_{Z}Y + {}^{4}_{2}He$ According to the law of conservation of the mass number: 210 = A + 4 ⇒ A = 206. According to the law of conservation of the charge number: 84 = Z + 2 ⇒ Z = 82. Thus: ${}^{210}_{84}Po \rightarrow {}^{206}_{81}Pb + {}^{4}_{2}He$	**1**
2.	Mass loss: $\Delta m = [m({}^{210}Po) - m({}^{206}Pb) - m({}^{4}He)]$· ¼ The liberated energy: $E_{lib} = \Delta m\cdot c^2$ ¼ $E_{lib} = [209.936800 - 205.929489 - 4.00151]\times931.5\ MeV/c^2\times c^2 = 5.404$ MeV. ½	**1**
3.	The liberated energy: $E_{lib} = E_C(Pb)+E_C(\alpha) = ½\, m_{Pb}V_{Pb}^2 + ½\, m\alpha V_\alpha^2$. ½ Conservation of the linear momentum: $\vec{P} = \vec{0} = m_{Pb}\,\vec{V}_{Pb} + m_\alpha\,\vec{V}_\alpha$; ½ Both speeds are in opposite directions. In absolute value: $m_{Pb}V_{Pb} = m\alpha V_\alpha$ et $V_{Pb} = V_\alpha\times m_\alpha/m_{Pb}$. ½ $E_C(Pb) = ½\, m_{Pb}V_{Pb}^2 = ½\, m_{Pb}\,(V_\alpha\times m_\alpha/m_{Pb})^2 = E_C(\alpha)\, m_\alpha/m_{Pb} = 0.01943\, E_C(\alpha)$ ¾ $E_{lib} = E_C(Pb)+E_C(\alpha) \Rightarrow 5.404 = 0.01943\, E_C(\alpha) + E_C(\alpha) \Rightarrow E_C(\alpha) = 5.30$ MeV ¾ $E_C(Pb) = 0.01943\times 5.303 = 0.103$ MeV.	**3**
4.1.	$\vec{v}_0 = v_0\,\vec{i}$; then $P_{0x} = mv_{0x} = 0$ and $P_{0y} = mv_0$. ½ The only external force applied is $\vec{F} = q_\alpha E\,\vec{i} = 2e\,E\,\vec{i}$. ½ According to Newton's second law: $\Sigma\vec{F} = d\vec{P}/dt = 2eE\,\vec{i}$. Ainsi : $dP_x/dt = 2eE$ et $dP_y/dt = 0$. (1½) By integration: P_y = constant $= m_\alpha v_y = m_\alpha v_0$ and $P_x = 2eE\,t + C_x$. $C_x = 0$ car $v_{0x} = 0$ and $P_x = 2eE\,t$; (1) thus a = 2eE et b = 0. ½	**4**
4.2.	$P_x = mv_x = 2eE\,t$ et $v_x = \frac{dx}{dt} = \frac{2eE}{m}t$ by integration: $x = ½\,\frac{2eE}{m}t^2 + D_x$. As $D_x = 0$, then $x = \frac{eE}{m}t^2$. (1) $v_y = \frac{dy}{dt} = v_0$ then by integration: $y = v_0\,t + D_y$. Since $D_y = y_0 = 0$ then $y = v_0\,t$. (1)	**2**
4.3.	We have $y = v_0\,t$ et $t = y/v_0$ consequently, the equation of the trajectory is: $x = \frac{eE}{mv_0^2}\,y^2$. ½	**0.5**
4.4.	En M, on a x = d et y = L ⇒ $d = \frac{eE}{mv_0^2}L^2$ et $v_0^2 = \frac{eE}{m\,d}L^2$. ½ Therefore: $KE(\alpha) = ½\, mv_0^2 = ½\frac{eE}{d}L^2 = ½\frac{1.602\times10^{-19}\times5\times10^{6}}{4.713\times10^{-3}}\times0.1^2 = 8.498\times10^{-13}$ J $KE(\alpha) = \frac{8.498\times10^{-13}}{1.602\times10^{-13}} = 5.30$ MeV(1½)	**2**

5.	The nucleus may be obtained in an excited state. It de-excites by releasing a γ photon.½ The energy of the photon: $E(\gamma) = 5.30 - 4.50 = 0.80$ MeV. ¾ $E(\gamma) = \frac{h\,c}{\lambda} \Rightarrow$ the wavelength is written as: $\lambda = \frac{h\,c}{E_\gamma} = \frac{6.626\times10^{-34}\times2.99792\times10^{8}}{0.80\times1.602\times10^{-13}} = 1.55\times10^{-12}$ m. (1)	2.25
7.	$N = \frac{m}{M}N_A = \frac{1}{210}\times6.022\times10^{23} = 2.87\times10^{21}$ Noyaux. ½ The radioactive constant λ is written as: $\lambda = \ln2/t_{½} = 0.693/(138\times24\times3600) = 5.812\times10^{-8}$ s^{-1}. ¾ The activity being given by $A = \lambda\cdot N$, then $A = 5.812\times10^{-8}\times2.87\times10^{21} = 166804\times10^{9}$ Bq or 166804 billion of becquerels. ¾	2
		21

Exercise II: Electromechanical energy converter: prototype

Q		Note
1.	The total flux Φ through (S) and (S ') is written: $\Phi = N\,\vec{B}\cdot\vec{A} + N\,\vec{B}\cdot\vec{A} = 2NBA\cos\theta = 2NBA\cos\omega t$. 1½	1.5
2.	According to Faraday's law: the induced electromotive force is given by: $e = -\frac{d\Phi}{dt} = 2NBA\omega\sin(\omega t)$ $\Rightarrow e = D\,\omega\sin(\omega t)$ with $D = 2NAB$. 1½	1.5
3.1.	By applying the law of addition of voltages, we obtain: $u_{AM} = u_{AB} + u_{BC} + u_{CM}$ soit : $e = 2ri + 2L\frac{di}{dt} + Ri$ (1) $D\omega\sin(\omega t) = (2r + R)\,I_m\sin(\omega t - \varphi) + 2L\omega I_m\cos(\omega t - \varphi)$ (1 ½) For $\omega t = 0$: $0 = (2r + R)\,I_m\sin(\varphi) + 2L\omega I_m\cos(\varphi)$ [1] ½ For $\omega t = \pi/2$: $D\omega = (2r + R)\,I_m\cos(\varphi) + 2L\omega I_m\sin(\varphi)$ [2] ½ The equation (1) $\Rightarrow \tan\varphi = \frac{2L\omega}{2r+R}$. ¾ As $\frac{1}{\cos^2\varphi} = 1 + \tan^2\varphi = 1 + \frac{(2L\omega)^2}{(2r+R)^2} = \frac{(2r+R)^2+(2L\omega)^2}{(2r+R)^2}$ therefore: $\cos\varphi = \frac{2r+R}{\sqrt{(2r+R)^2+(2L\omega)^2}}$. (1) $\sin\varphi = \cos\varphi\cdot\tan\varphi = \frac{2L\omega}{\sqrt{(2r+R)^2+(2L\omega)^2}}$. ¾ We have: $D\omega = (2r + R)\,I_m\cos(\varphi) + 2L\omega I_m\sin(\varphi) = (2r + R)\,I_m\frac{2r+R}{\sqrt{(2r+R)^2+(2L\omega)^2}} + I_m\frac{2L\omega}{\sqrt{(2r+R)^2+(2L\omega)^2}}$ $D\omega = \frac{I_m}{\sqrt{(2r+R)^2+(2L\omega)^2}}[(2r + R)^2 + (2L\omega)^2]$. Enfin : $I_m = \frac{D\omega}{\sqrt{(2r+R)^2+(2L\omega)^2}}$ (1) Which gives: $i = \frac{D\cdot\omega}{\sqrt{(2r+R)^2+(2L\omega)^2}}\sin(\omega t - \varphi)$. ½	7.5
3.2.1.	Referring to document 3: T = 0.4 s ½ which gives the angular frequency ω of the rotor: $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.4} = 5\pi = 15.7$ rad/s. (1)	1.5
3.2.2.	The average power P delivered by the coils (S) and (S') to (R) is written: $P = R\cdot I^2$, where $I = \frac{I_m}{\sqrt{2}} = \frac{0.5}{\sqrt{2}} = 0.354$ A. ½ ½ Thus, $P = 2\times0.354^2 = 0.25$ W. ½	1.5
4.1.	The mechanical power P_m received by the alternator is written as follows: $P_m = M_m\cdot\omega = 3.8\times10^{-2}\times15.7 = 0.597$ W. (1)	1
4.2.	The efficiency can be written as: $\rho = P/P_m = 0.25/0.597 = 0.419 \approx 42\%$. (1)	1
4.3.	We have: $P_m = P + P_{Jint} + P_f$. $P_f = P_m - P_{Jint} - P = 0.597 - 0.25 - 0.25 = 0.097$ W. Since $P_f = M_f\cdot\omega$. Then: $M_f = P_f/\omega = 0.097/15.7 = 0.062$ m·N.	1.5

5.	We have : $I_m = \frac{D\cdot\omega}{\sqrt{(2r+R)^2+(2L\omega)^2}} \Rightarrow 0.5 = \frac{D\times 15.7}{\sqrt{(2\times1+2)^2+(2\times0.2\times10^{-3}\times15.7)^2}} = 3.925\ D \Rightarrow$ D = 0.5/3.925 = 0.127 Wb.	**1**
		18

Exercise III : Torsion pendulum

Q		Note
A. 1.1.	The moment of inertia I_Δ of (Σ) with respect to the axis (Δ) is written as: $I_\Delta = I_0 + 2mL^2 = \frac{1}{3}m_T\cdot L^2+2mL^2 = (\frac{1}{3}m_T+2m)L^2$ (1) $I_\Delta = (\frac{1}{3}0.090+2\times0.05)0.1^2 = 1.30\times10^{-3}$ kg m². ¾	**1.75**
1.2.	The mechanical energy of the system (pendulum, Earth) is written: $E_m = ½\ I_\Delta\dot\theta^2 + ½C\theta^2$ = constant. (1) $\frac{dE_m}{dt} = I_\Delta\dot\theta\ \ddot\theta + C\theta\ \dot\theta = 0$. ½ $I_\Delta\dot\theta(\ddot\theta + \frac{C}{I_\Delta}\theta) = 0$. But $\dot\theta$ is not always zero, so we obtain the differential equation: $\ddot\theta + \frac{C}{I_\Delta}\theta = 0$. (1)	**2.5**
1.3.	The differential equation is of the form: $\ddot\theta + \omega_0^2\ \theta = 0$. By identification we obtain: $\omega_0^2 = \frac{C}{I_\Delta}$ ¾ The proper angular frequency will have for expression: $\omega_0 = \sqrt{\frac{C}{I_\Delta}}$. ½ The expression of the proper period T_0 is given by: $T_0 = \frac{2\pi}{\omega_0}$ and $T_0 = 2\pi\sqrt{\frac{I_\Delta}{C}}$. ¾	**2**
2.1.	The frictional forces are the only forces that affect the motion of the pendulum. ¼ The power $P = \frac{dE_m}{dt} = M_f\cdot\dot\theta = -2\lambda\omega_0\dot\theta^2 \Rightarrow -2\lambda\omega_0\dot\theta^2 = I_\Delta\dot\theta\ \ddot\theta + C\theta\ \dot\theta$. (1) Thus, $I_\Delta\dot\theta(\ddot\theta + \frac{2\lambda\omega_0}{I_\Delta}\dot\theta + \frac{C}{I_\Delta}\theta) = 0$. Or $\dot\theta$ is not always zero, So: $\ddot\theta + \frac{2\lambda\omega_0}{I_\Delta}\dot\theta + \frac{C}{I_\Delta}\theta = 0$. (1)	**2.25**
2.2.1.	The pendulum follows a pseudo-periodic regime and the damping is weak because the amplitude of the oscillations decreases weakly with each oscillation.	**0.5**
2.2.2.	Referring to document 2, we find that 12 T cover 3500 s, then: T = 3500/12 = 292 s. ¾ The pseudo-angular frequency is written as: $\omega = \frac{2\pi}{T} = \frac{2\pi}{292} = 0.0215$ rad/s. ¾ $\tan\varphi = \frac{\lambda}{\sqrt{1-\lambda^2}} = \frac{0.25}{\sqrt{1-0.25^2}} = 0.258 \Rightarrow \varphi = 0.253$ rad. ¾ At the instant $t_0 = 0$, $\theta_0 = 0.080 = A\cos\varphi \Rightarrow A = \frac{0.080}{0.968} = 0.083$ rad. ¾ We have : $\omega_0 = \frac{\omega}{\sqrt{1-\lambda^2}} = \frac{0.0215}{\sqrt{1-0.25^2}} = 0.0222$ rad/s. ¾ Since $\omega_0^2 = \frac{C}{I_\Delta}$ and $C = I_\Delta\cdot\omega_0^2 = 1.30\times10^{-3}\times0.0222^2 = 6.41\times10^{-7}$ SI. (1)	**4.75**
B.1.1	At the instant t, (Σ) makes an angle θ with its equilibrium position, then the ball (S_1) is at a distance s = Lθ of its initial equilibrium position. ½ Thus, at an instant t, the distance between the sphere (S_2) and the ball (S_1) is r = d - Lθ ½	**1**
B.1.2	The expression of the gravitational force between (S_1) and (S_2) is: $F = \frac{GMm}{r^2} = \frac{GMm}{(d-L\theta)^2}$. ½ At the instant t, the system (Σ)is submitted to - the tension $\vec{T}$ of the wire of zero moment: $M_T = 0$; ½ - the torsion torque of moment: $M_1 = -C\theta$; ¼	**2.5**

	- the torque of friction forces of moment: $M_2 = -2\lambda\omega_0\dot{\theta}$; ¼ - the torque of de gravitational forces $(\vec{F}, \vec{F'})$ of moment with respect to (Δ) : $M_3 = 2FL\cos\theta \approx 2FL = \frac{2LGMm}{(d-L\theta)^2}$. (1)	
2.	By applying the angular momentum theorem, we obtain: $M_T + M_1 + M_2 + M_3 = \frac{d\sigma}{dt} = I_\Delta\ddot{\theta}$. (1) Therefore, the differential equation in θ is: $\frac{2LGMm}{(d-L\theta)^2} - C\theta - 2\lambda\omega_0\dot{\theta} = I_\Delta\ddot{\theta}$, which describes the motion of (S). ¾	**1.75**
3.	Referring to Doc 4, the equilibrium position is for $\theta_e = 1$ mrad $= 1\times10^{-3}$rad. ½ In this case $\dot{\theta} = 0$ et $\ddot{\theta} = 0$. ½ $\frac{2LGMm}{(d-L\theta)^2} - C\theta = 0$ and $G = \frac{C\theta_e(d-L\theta)^2}{2LMm} = \frac{6.41\times10^{-7}\times1\times10^{-3}(0.177-0.1\times1\times10^{-3})}{2\times0.1\times0.05\times30} = 6.7\times10^{-11}$ SI. (1)	**2**
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