UNIVERSITÉ LIBANAISE
Faculté des Beaux-Arts et d'Architecture
- Le doyen -

الجامعة اللبنانية
كلية الفنون الجميلة والعمارة
– العمادة –

مباراة الدخول الى معهد الفنون الجميلة للعام 2019-2020

قسم الهندسة المعمارية | **مسابقة في الرياضيات (انكليزي)** | **المدة: ساعتان**

All CALCULATORS are prohibited.

Exercise I. CHOOSE A UNIQUE CORRECT ANSWER WITH JUSTIFICATION. ▶ **10 × 1.5** points

Write down the correct answer cleary and [circle] it. Then, justify your answer. *Evrey answer without justification will not be taken into consideration.*

1. In the space of Oxyz, given a plane (P) : $2x-2y+z-1=0$ and a straight line (D) : $\begin{cases} x = \frac{t}{2} - 1 \\ y = 1 - t \\ z = t + 2, \end{cases}$
   with t real parameter. The angle made by (D) with the plane (P) is equal to :
   ▶ $\arcsin \frac{8}{9}$ ▶ $\arcsin \frac{17}{9}$ ▶ $\arccos \frac{8}{9}$ ▶ $\arcsin \frac{9}{8}$.

2. An urn contains 10 balls numbered from 1 to 10. If three balls are drawn simultaneously at random, then the probability that the numbers of the obtained balls are such that one is odd and the other two are even, is :
   ▶ $\frac{1}{8}$ ▶ $\frac{5}{12}$ ▶ $\frac{5}{6}$ ▶ $\frac{6}{5}$.

3. Consider the real function $g(x) = \dfrac{1}{1+e^{-x}}$. Studying its variation, we conclude that g is :
   ▶ strictly increasing on $\mathbb{R}$, and admits two horizontal asymptotes $y = 0$ and $y = 1$.
   ▶ increasing on $\mathbb{R}^+$, and admits two asymptotes $y = \frac{1}{2}$ and $y = 0$.
   ▶ strictly increasing on $\mathbb{R}$, and admits two asymptotes $y = 0$ and $x = 0$.

4. Consider, in $\mathbb{C}$, the equation (E) : $z^3 - (1+2i)z^2 + (1+i)z - 2(1+i) = 0$.
   If one root of (E) is $z_1 = 1 + i$, then the other two roots, z_2 and z_3, are equal to :
   ▶ $z_2 = -i, z_3 = i + 2$ ▶ $z_2 = i, z_3 = 2i$ ▶ $z_2 = -i, z_3 = 2i$ ▶ $z_2 = z_3 = -i$.

5. Given the two planes, (P) : $x + y + z = 1$ and (Q) : $2x - y + 3z = 0$. Let (R) be the plane passing through the point A(1,1,1) and perpendicular to both planes (P) and (Q). The equation of (R) is :
   ▶ $-4x+y+3z+1=0$ ▶ $4x-y-3z=0$ ▶ $2x-y-z=0$ ▶ $4x-y+3z=0$.

6. Consider the complex number $z = x + iy$; $(x, y) \in \mathbb{R}^2$, with M its image in the Oxy plane. Let $Z = \dfrac{z - \overline{z} + 2}{z + 1 - i}$. If Z is real, then the set of M is the :
   ▶ circle $(x+1)^2+(y-1)^2 = 1$ ▶ line $y = -x$ ▶ hyperbola $y = -\frac{1}{x}$ ▶ hyperbola $xy = 1$.

7. If a weighted cubic die is made in such a way that, when it is tossed, **the probability of the number 1 is 0.8**, and the other five numbers from 2 to 6 are equiprobable, then on throwing this die once, the probability of obtaining an odd number is :
   ▶ 0.88 ▶ 0.80 ▶ 0.84 ▶ 0.82.

8. The following real function $f(x) = \ln(\ln(\ln x^2))$ is defined on :
   ▶ $]-\infty, -1[\cup]1, +\infty[$ ▶ $\mathbb{R} \setminus \{-\sqrt{e}, -1, 0, 1, \sqrt{e}\}$ ▶ $]-\infty, -\sqrt{e}[\cup]\sqrt{e}, +\infty[$ ▶ $\mathbb{R}^*$.

9. If $x > 0$ and $J(x) = \int_x^{x+1} \frac{dt}{1+t}$, then $J(x)$ belongs to the interval :
▶ $]\frac{1}{x+2}, \frac{1}{x+1}[$ ▶ $]\frac{1}{x+1}, \frac{1}{x}[$ ▶ $]x, x+1[$ ▶ $]0, +\infty[$.

10. Let n be an **even** natural number and $I_n = \int_0^{\pi} (\sin^n t)\, dt$. If we have the recurrent relation $nI_n = (n-1)I_{n-2}$, then I_0 and I_6 are equal to :
▶ $I_0 = \pi, I_6 = \frac{5\pi}{6}$ ▶ $I_0 = \pi, I_6 = \frac{15\pi}{48}$ ▶ $I_0 = \pi, I_6 = \frac{15\pi}{24}$ ▶ $I_0 = \pi, I_6 = \frac{5\pi}{48}$.

Exercise II. CHOOSE THE UNIQUE CORRECT ANSWER WITHOUT JUSTIFICATION. ▶ 25 points
Copy the table, according to the model below, to your answer sheet and fill in the correct answer of each question without justification.

Number of Question	Chosen answer
Q1	
Q2	
⋮	
Q25	

Q1. The plane $3x + y + z - 1 = 0$ and the straight line $\frac{x}{2} = \frac{1-y}{2} = \frac{-z}{2}$ are :
▶ parallel ▶ perpendicular ▶ intersecting ▶ confounded.

Q2. The parabola $y^2 = 4x - 1$ admits a vertex at the point :
▶ $(\frac{1}{2}, 0)$ ▶ $(2, 1)$ ▶ $(\frac{1}{4}, 0)$ ▶ $(2, 3)$.

Q3. $x^2 + y^2 = 2x$ is a circle of radius equals to :
▶ -1 ▶ $\frac{\sqrt{2}}{2}$ ▶ $\sqrt{2}$ ▶ 1.

Q4. Given the sequence of real numbers $\frac{-1}{4}, \frac{1}{7}, \frac{3}{10}, \ldots$ Then, the next term is :
▶ $\frac{4}{11}$ ▶ $\frac{4}{13}$ ▶ $\frac{5}{12}$ ▶ $\frac{5}{13}$.

Q5. $\int \frac{\cos x}{2\sin x - 3} dx$ equals to :
▶ $\frac{1}{2}\ln(2\sin x - 3) + C$ ▶ $\frac{1}{2}\ln|2\sin x - 3| + C$ ▶ $\frac{1}{2}\ln|2\sin x| - 3$ ▶ $\ln|2\sin x - 3| + C$.

Q6. The straight line $\frac{x}{2} = \frac{y-1}{3} = \frac{z+1}{1}$ and the z-axis are :
▶ skew ▶ intersecting ▶ parallel ▶ orthogonal.

Q7. If we join the mid points of a square whose side is 6 cm, then the area of the new obtained square is :
▶ 9 cm^2 ▶ 12 cm^2 ▶ 16 cm^2 ▶ 18 cm^2.

Q8. One angle of a triangle is $30°$, and the difference between the other two angles is $70°$, then the largest angle in the triangle is :
▶ $70°$ ▶ $110°$ ▶ $120°$ ▶ $40°$.

Q9. Given the sequence $-3, 0, 3, 6, 9, \ldots$ then the sum of the first ten terms is :
▶ 110 ▶ 105 ▶ 95 ▶ 90.

Q10. $\lim_{x \to +\infty} \frac{\sqrt{x^2 - 2}}{\sqrt[3]{x^3 + 3}} =$
▶ 1 ▶ $\frac{2}{3}$ ▶ 0 ▶ -2.

Q11. $\lim_{x \to 0} \frac{\sin^2 \frac{x}{2}}{x^2} =$

▶ $+\infty$ ▶ 1 ▶ indeterminate ▶ $\frac{1}{4}$.

Q12. The non trivial general solution of the differential equation $y' - y \sin x = 0$ is :

▶ $Ce^{\sin x}$ ▶ $Ce^{\cos x}$ ▶ $Ce^{-\cos x}$ ▶ 0.

Q13. The function $y = x^x$ is increasing over the interval :

▶ $]0, +\infty[$ ▶ $]e, +\infty[$ ▶ $]\frac{1}{e}, +\infty[$ ▶ $[\frac{1}{e}, +\infty[$.

Q14. A student has to answer 10 out of 13 questions on an exam. How many choices he has, if he **must** answer the first two questions :

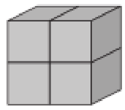
▶ C_{13}^{10} ▶ C_{11}^{8} ▶$C_{11}^{8} \cdot C_{11}^{2}$ ▶ C_{13}^{8}.

Q15. The function $y = \frac{\sqrt{1-x}}{x-2}$ admits as asymptotes :

▶ $x = 0; y = 0$ ▶ $y = 0$ only ▶ $x = 2; y = 0$ ▶ $x = 2; y = 2$.

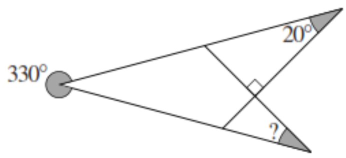
Q16. The sketched solid is formed of four small identical cubes. The surface area of a small cube is 24 cm^2. What is the surface area of the solid ?

▶ 80 cm^2 ▶ 64 cm^2 ▶ 40 cm^2 ▶ 32 cm^2.



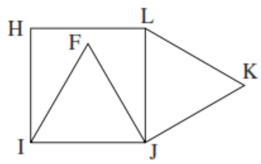
Q17. With the data in the opposite figure, how much the unknow angle (?) would be equal to ?

▶ $10°$ ▶ $40°$ ▶ $20°$ ▶ $30°$.



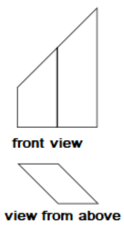
Q18. HIJL is a square. The triangles IJF and JKL are equilateral. If HL=1, then what would be the segment FK equal to ?

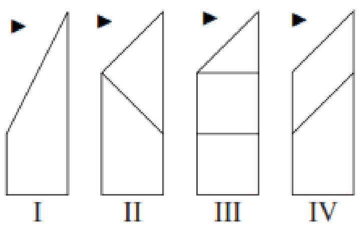
▶ $\sqrt{2}$ ▶ $\frac{\sqrt{3}}{2}$ ▶ $\sqrt{3}$ ▶ $\sqrt{6} - 1$.



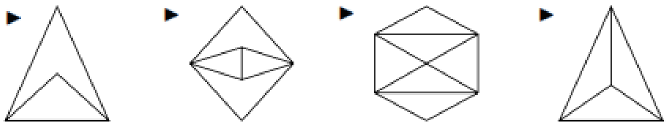
Q19. The figure below represents the front view of a solid, and its view from above. Among the four proposed figures I, II, III and IV, which one represents the solid seen from the left ?

▶ la figure I ▶ la figure II ▶ la figure III ▶ la figure IV.



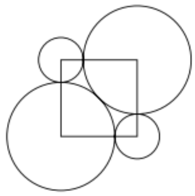


Q20. Which one among the following four figures cannot be drawn without raising the pencil and without tracing twice any of the segments ?

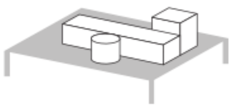


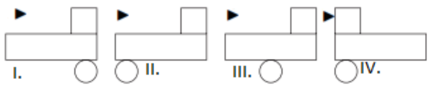
Q21. Consider two great circles tangent to each other with radius $\boldsymbol{R}$ and with centers at two opposite vertices of a square. With centers at the other two vertices, two small circles of the same radius $\boldsymbol{r}$ are traced such that each circle is tangent to the two great circles. How much is the ratio $\dfrac{\boldsymbol{R}}{\boldsymbol{r}}$?

▶ $\mathbf{2}$ ▶ $\sqrt{\mathbf{5}}$ ▶ $\mathbf{1}+\sqrt{\mathbf{2}}$ ▶ $\mathbf{0.8\pi}$.



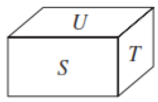
Q22. There are three objects on the table. What do you see when looking at the table from above ?





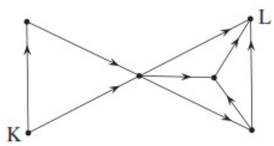
Q23. The areas of the faces of a rectangular brick are $\boldsymbol{S}$, $\boldsymbol{T}$ and $\boldsymbol{U}$ (see figure). What is the volume of the brick ?

▶ $\boldsymbol{STU}$ ▶ $\sqrt{\boldsymbol{STU}}$ ▶ $\sqrt{\boldsymbol{ST}+\boldsymbol{TU}+\boldsymbol{US}}$ ▶ $\sqrt[3]{\boldsymbol{STU}}$.



Q24. One have to go from K to L following the arrows (see figure). How many differents routes can be used ?

▶ **6** ▶ **8** ▶ **9** ▶ **10**.



Q25. Eight semi-circles of the same radius are drawn in a square of side 4 (see figure). How much is the area left blank in the square ?

▶ $\mathbf{2\pi}$ ▶ $\mathbf{8}$ ▶ $\boldsymbol{\pi}+\mathbf{6}$ ▶ $\mathbf{3\pi-2}$.