الجامعة اللبنانية — مباراة الدخول للعام الجامعي 2024-2025

كلية الاعلام — مسابقة في الرياضيات -قسم علم البيانات – إنكليزي

مدة الامتحان: ساعة ونصف

**Answer the following questions:**

**Exercise 1:**

**Part A:**

Consider the function g defined on the interval ]-∞, +∞[ by $g(x) = xe^x - 1$

1. Calculate the limits of g at the open bounds of its domain.
2. Calculate g'(x) and then draw up the variation table of g.
3. a) Prove that the equation g(x) = 0 has a unique root α on ℝ and verify that: $0.5 < \alpha < 0.58$.

   b) Deduce the sign of g(x).

**Part B: Analyzing function f(x)**

Let f be the function defined on ]-∞, +∞[ by $f(x) = (x-1)(e^x-1)$, Let (C) be its representative curve in an orthonormal coordinate system (o, i, j) (unit of length = 2cm).

1. a) Determine the limit of f(x) as x tends to +∞ and -∞, and show that the line (d): y =-x+1 is an asymptote to (C).

   b) Study the relative position of the curve (C) with respect to the line (d).

2. Determine the $\lim_{x \to +\infty} f(x)$ and calculate f(2).
3. Prove that f'(x) = g(x) and then draw up the variation table of f. (We assume f'(x) < 0 for $x < \alpha$).
4. Show that the function f has an inflection point I whose coordinates are to be determined.
5. (T) is parallel to (d) and tangent to (C) at A. Calculate the coordinates of A.
6. Verify that $f(\alpha) = 2 - \alpha - \frac{1}{\alpha}$
7. Take α = 0.57 and draw (d) and (C).

**Exercise 2:**

The table below is a model that gives the blood pressure of a group of ladies, as well as their weight.

| Weight in Kg, $x_i$ | 55 | 58 | 60 | 64 | 65 | 70 |
|---|---|---|---|---|---|---|
| Blood pressure, $y_i$ | 13.2 | 13.5 | 13.8 | 14.6 | 15.2 | 15.8 |

1. Calculate the means $\bar{x}$ and $\bar{y}$
2. Represent the scatter plot of points in an orthonormal system, and the mean point G.
3. a. Calculate the covariance of the variables $x_i$ and $y_i$.
   b. Calculate the variances of the variable $x_i$ and of the variable $y_i$ .
   c. Calculate the correlation coefficient of the variables $x_i$ and $y_i$ and interpret the result.
4. a. Find the equation of the regression line $D_{Y/X}$ of $Y$ in term of $X$ .
   b. Draw $D_{Y/X}$ in the preceding orthonormal system.
5. If this model is true for ladies weighting between 45 and 75 kg, estimate the blood pressure of a lady weighting 72 kg.
6. The doctors assume that a normal blood pressure must belong to the interval [12; 13]. Estimate an interval to which the weight of a lady must belong, if her blood pressure is normal.

**Exercise 3:**

An urn contains 3 fair coins. Two of them are normal: they have a "Heads" side and a "Tails" side. The third is rigged and has two "Heads" sides.

A coin is taken at random from the urn and successive tosses of this coin are performed independently. Consider the following events:

$B$: the coin taken is normal. $\bar{B}$: the coin taken is rigged.

$P$: we obtain "Heads" on the first toss.

$F_n$: we obtain "Tails" for the first n tosses.

1) What is the probability of event $B$?
2) What is the probability of event $P$ given that $B$ occurs?
3) Calculate the probability of event $P \cap B$, then of event $P \cap \bar{B}$. Deduce the probability of event $P$.

Calculate the probability of the event $F_n \cap B$ then of the event $F_n \cap \bar{B}$. Deduce the probability of the event $F_n$.